Variational rationality, routines and the "unsatisfied man": the course pursuit problem between aspirations, capabilities and beliefs

Antoine Soubeyran, GREQAM

December 15, 2010

Abstract

This paper modelizes the "unsatisfied man" who enters in a course pursuit between ends and means, feeling an unsatisfaction- aspiration gap, setting goals (satisficing levels), gathering resources and building capabilities to partially complete the present aspiration gap. More precisely, each step, an agent moves from a temporary routine to a new one to fulfill a satisficing portion of his aspiration gap, using some worthwhile change which balances, each step, motivation and resistance to change. Extending the famous March "exploration-exploitation" model, we propose a three stage model of change, where each period, to overcome his unsatisfaction, the agent considers (aspires, sets goals, imagines, explores and evaluates), implements and exploits change. We give conditions under which, along a path of worthwile changes, the aspiration gap vanishs and temporary routines converge to a permanent routine (an aspiration trap) where the agent prefers to stay than to move. We show how our variational approach proposes a general theory of (stability and) change which explains how temporary routines intentionally change, in a more or less punctuated way, the evolution of aspiration and satisficing levels, beliefs and capabilities playing a major role. This paves the way to micro foundations of behavioral and capability-evolutionary economics as well as to the psychology of change (theories of motivation and inertia). At the mathematical level the famous variational principle of Ekeland appears as a special case.

JEL Classification number: C.61, C.52, D.03, D.83, 0.31

Keys words: action, capability, temporary routine, aspiration gap, satisficing gap, worthwhile change, aspiration trap, convergence.

1 Introduction

Human life is a mix between stability and change: how routines intentionally change ? Every life is made of a succession of long or short periods of stability (temporary routines) and long or short periods of changes, where, from time to time, punctually or gradually, agents and organizations break temporary habits and routines (actions repeated several times) to move to new ones. These changes are more or less driven by intermediate goals, in a goal pursuit towards a final goal which stops to motivate change as soon as it have been reached, a behavioral trap. Then, every life is a course pursuit between means and ends where preferences change more or less with experience coming from the succession of past actions. Agents make new actions based on present preferences, which will change their future preferences, which in turn lead to choose to do new actions...As a consequence, isolated or interacting agents will be, most of the time, out of an optimum, an equilibrium or a rest point. Then, how can they reach one of them, starting from somewhere ?

This paper modelizes the "unsatisfied man" who enters in a course pursuit between ends and means, feeling an unsatisfaction-aspiration gap (the difference between an aspiration level and the present performance or payoff level). Notice that an aspiration level is not a satisficing level (Simon, 1955). It represents a self estimation of the highest goal level the agent can reach, a belief about a supremum (see later). Then, either the agent, being depressed, renounces to change, or has the intention to change. In this pro-active case he sets goals (satisficing levels), gathers resources and builds capabilities to reach his satisficing level which partially completes the present aspiration gap. More precisely, each step, an agent moves from a temporary routine to a new one to fulfill a satisficing portion of his aspiration gap, using some worthwhile change which balances, each step, motivation and resistance to change. Extending the famous March (1991) "exploration-exploitation" model, we propose a three stage model of change, where each period, the agent, to overcome his unsatisfaction, considers (aspires, sets goals, imagines, explores and evaluates), implements and exploits change. We give conditions under which, along a path of worthwile changes, the aspiration gap vanishs and temporary routines converge to a permanent routine (an aspiration trap) where the agent prefers to stay than to move. Our variational approach proposes a general theory of (stability and) change which explains how, in a more or less punctuated way, temporary routines intentionally change. We show how the evolution of aspiration and satisficing levels, beliefs and capabilities play a major role. This paves the way to micro foundations of behavioral economics (Simon, 1955) and capability-evolutionary economics (the resource based theory of the firm, see Mahoney, 2005) as well as to the psychology of change (theories of motivation and inertia). At the mathematical level the famous variational principle of Ekeland appears as a special case.

To be happy with something: the utility theory of choice The static theory of behaviors emphasizes situations where people are happy with something. To be happy leads to a theory of utility (contentment, satisfaction), where the goal is to directly consider situations where agents prefer to stay than to move (optimum, equilibrium), without paying attention to understand how they reached this situation. This is the essence of optimization models (substantive rationality). The main example is a problem of choice between a given list of alternatives which defines the given considered subset of choice. In this case nothing change and the whole internal and external environment is known The agent has a stable preference relation and an initial global view of the situation. He is aware of the existence of the whole subset of alternatives and he knows, ex ante, the utility of each alternative. To find the global optimum, the agent uses the so called "dichotomy principle". In a first (and hidden) stage, he discovers the whole consideration set of alternatives and the utility of each of them. In a second stage he tries to find the optimum. He compares alternatives pair by pair, deleting the least prefered of each pair, until he has exhausted the whole subset of alternatives. Then, the (stopping) time spent to compare all pairs of alternatives does not depend of the choice of the initial pair and this successive elimination process of alternatives does not depend of a statu quo. There are no costs to optimize, or when costs to optimize exist, they are known ex ante. Hence the agent does not enter in the famous "infinite regression problem" to know the costs to know the costs ....of search and to choose "what to ignore, ignoring the consequences". Preferences do not change (they do not depend of a variable reference point). Experience and tacit knowledge do not matter. It is "as if" the model has one stage. To be realistic enough, this requires a small state space of alternatives, and a short enough list of criteria to estimate the utility of each alternatives.

To be unhappy with something: theories of change A theory of change starts from a present situation, a statu quo from which agents can choose to stay or to move. It considers situations where agents are "unhappy with something", feel a frustration gap, have unsatisfied needs, aspire to more, and have the intention to change (want rather than renounce to change). The emphasize is more on the unhappiness to be in a given situation than to the happiness to be there. Agents set aspiration levels which impluse their desire to change. They enter in a transition phase, made of a succession of acceptable changes with not too much temporary sacrifices, where they build, step by step, new capabilities. Final positions are aspiration traps where all agent, being unhappy of nothing, prefers to stay than to move (their aspiration gaps have vanished).

The simplest example of a model of change is the following. Consider a consumer whose utility function is $g(x) = x/(1+x)$ and $x \geq 0$ is his consumption level. Suppose that the agent is unhappy with the statu quo utility $g(x_0) \in]0, 1[$ of his initial consumption level $x_0 > 0$. Then, the consumer has in mind some higher aspiration level, $\widehat{g} > g(x_0)$ and is unhappy of his aspiration gap $\widehat{a}(x_0) = \widehat{g} - g(x_0) > 0$. If he is pro-active, he wants to change his consumption level from x_0 to x to improve his utility from $g(x_0)$ to $g(x) > g(x_0)$. This will reduce his frustration-aspiration gap from $\widehat{a}(x_0) > 0$ to $\widehat{a}(x) > 0$. The aspiration level $\widehat{g}$ of the agent can be reachable or not, being, lower, equal or higher than the supremum $\overline{g} = 1$ of the utility function.

The main questions and problems of the theories of change **Main questions**. Schumpeter (1950) is the grand father of the theory of change. For him, entrepreneurs implement change and move from old to new. More generally the mains questions of any theory of change, driven by aspiration-frustration gaps are,

1) "what to change": actions (efforts, repeated actions like habits and routines), goals (performances, payoffs, revenue minus costs "to do", utilities), capabilities (resources, skills, competences, knowledge, abilities to do), internal states (health, wealth, beliefs and expectations, sense of self efficacy, self esteem, emotions, like pleasure and pain, contentment and frustration, arousal and regret, intentions, attitudes....), contexts (external environment),.... behaviors ("stimuli-response" functions, what to do in given circumstances).....

2) "why to change" (intentions and motivations to change).

3) "why to do not change" (resistances to change).

4) "where to change" (setting aspiration levels or final goals).

5) "how to change" (searching for an acceptable path of change), setting intermediate goals (temporary satisficing levels).

6) to prepare and implement change (building capabilities, learning).

7) "when to change" (choosing the speed and timing of change, using an incremental, gradual or punctuated dynamic).

Problems of change. They abound (the only thing which does not change is change !). Examples are production and transformation problems, like creation, invention, innovation, search, sequential choice problems of elimination, goal pusuit, learning problems, habits and routines formation, convergence to a Nash equilibrium, evolution of interactions between agents, belief's formation, ..., organizational changes, problems of transitions, institutional changes, cultural changes, life happiness.....

The course pursuit problem. When an agent, being unhappy with a situation, imagines a subset of new actions which can allow him to decrease his frustration feeling and fill a portion of his aspiration gap, the agent must acquire new capabilities (learn and be able to do). Before "doing" a new action, he must be "able to do it". Then, the agent enters in a course pursuit between motivation and learning problems related to experience, aspirations, beliefs, perceptions, goals as frames, actions and capabilities. The agent must solve in an adaptive way,

1) learning problems (search and capabilty building). How, each step, to change the state space of considered actions (for production problems) and alternatives (for pure choice problems)? Which new actions must be considered, which old actions must be deleted or conserved (the state space is generated each step, following evolutionary economics where it is not a given) ? How to build the capabilities to do new actions and how to destroy or maintain, available for use, the capabilities to do old actions?

2) motivation problems (goal setting, goal striving). How habits and routines intentionally change? How agents set and complete their goals, setting intermediate goals, like temporary satisficing levels within a moving interval of frames where the lower bound is the present utility and the upper bound is the temporary aspiration level. How they use feed backs like successes and failures to adapt their goals ?

In this context where everything change, preferences change step by step with experience and the levels of performance which has been realized, using intermediate goals as frames ? Psychological and cognitive aspects (emotions, learning, personality traits) matter...

Procedural rationality and variable preferences: transition matter For us, procedural rationality (Simon, 1955) is essentially an "out of equilibrium" theory which advocates that it is more economizing to follow "improving from time to time" paths, instead of optimizing each step. Let us show why, in a variational approach which focuses attention to acceptable transitions from an initial position to a behavioral trap where motivations to change has disappeared (an optimum, an equilibrium, a rest point, , ...see the definitions later), agents will prefer to follow such economizing paths, satisficing without too much sacrificing, using worthwhile changes (as we will see later, this is a way to define them).

1) Transitions matter, because, to solve complex problems, agents cannot escape from them. There is no way to solve these problems in one step. The agent has to divide the difficulty, to bracket the problem to economize on convex costs and costs to carry out actions quickly. Furthermore, each step, time energy and other resources are limited.

2) Preferences change during the transition. Then, it is more economizing, each step, to explore locally. Suppose the contrary. To be able to optimize each step, the agent would have to repeatedly explore the whole (partially new) state space to discover the entire new utility function before exploiting the choosen new optimal action. To estimate the whole new utility function will require to spend too much of the limited time and resources, given that, next step, preferences will have changed. Then, choosing, each step, an optimizing action instead of an improving one would be too expansive (too much exploration compared to exploitation).

3) A transition must be acceptable. Transitions generates temporary sacrifices, exploration costs, costs to set intermediate goals, and costs to build capabilities. Then, the agent must overcome these resistances to change. Inertia matters. To sustain his motivation, the agent will try, during the transition, to limit his temporary sacrifices and to temporary satisfice enough. Being obliged to entwin, each step, exploration (which is costly) and exploitation (which is rewarding), he must balance them efficiently.

These remarks justify in an original way the bounded rational critics of Simon (1955). They question one of the main hypothesis in economics, which makes "as if" agents optimize. In this paper our problem is to define variable degree of bounded rationality for different agents, between two extremes. The first extreme is the "muddling through" hypothesis (improving, making small steps) which is standard in political sciences (Lindblom, 1959). The other extreme is the global optimization hypothesis, which is standard in economics, where the main problem is to verify that, "being there, without knowing why he is there (a paradox for applied sciences !), the agent prefers to stay there than to move". This is a "stability" problem which focuses on a given point, and necessary and sufficient conditions to be an equilibrium. Between these two extremes, "variational problems" examine "transition problems" which focuse attention on, i) why agents "are there", locked in a behavioral trap ?, or ii) where agents will try to go ?. The problem is to know when and how, starting from some point, out of a behavioral trap, an isolated agent or several interacting agents will reach a behavioral trap, using worthwhile changes and more generally different kinds of acceptable changes. Three examples of transition problems are "learning how to reach an optimum", " learning how to play a Nash equilibrium" (Chen-Gazzale, 2004) and "organizational change" (the centtral problem of evolutionary economics, see Soubeyran. A, 2010a) . Our paper limits the analysis to one agent.

Goal pursuit and the "exploration-exploitation trade off". To materialize these intuitions, this paper revisites the famous "exploitation-exploration" learning model of change (Holland, 1975, March, 1991) in the context of the "goal pursuit problem", where an agent, "being somewhere, wants to go there". Then, he sets intermediate goals, following acceptable transitions, using worthwhile changes which compensate, each step, some "resistance to change" by a high enough "motivation to change". Exploration and exploitation represent one aspect of resistance and motivation to change. These changes generate variable preferences driven both by i) temporary goals working as frames (reference points), ii) related emotions coming from anticipated and realized changes, and iii) the new capabilities that the agent must built, each step, to hope to be able to reach these temporary goals. Then, the agent enters in a course pursuit between means and ends (Soubeyran. A, 2010b) to eventually reach a behavioral trap where he prefers to stay than to move, stopping to build new capabilities and learning, because his motivation to change does not "compensate enough" his resistance to change.

An "aspiration gap" model of change. We propose a model of change where, each period, the agent starts from the current state, a temporary routine. This is a temporary reference point which is a temporary statu quo point, the agent being able to stay there, spending some maintenance costs. The agent, having in mind some temporary aspiration level driven by his ambition and his past experience, compares his present temporary payoff to his temporary aspiration level. The difference, if any, defines a temporary aspiration gap, supposed to be non negative. This gap generates some frustration feelings. If the aspiration gap is zero, the agent has no motivation to change and will stay there, having reached an aspiration trap.

A model of satisficing changes. Suppose now that the temporary aspiration gap is strictly positive. Then, either the agent renounces to change or has the intention to change. In this later case he enters a goal pursuit process, moving from a temporary routine to a new improving one, in order to decrease his frustration feelings. One way to do that is to start a gradual satisficing process. The agent can try to fill, each step, some minimal portion of his temporary aspiration gap. Each step, he can set a temporary satisficing level which defines a temporary satisficing gap, the portion of his temporary aspiration gap he wants, at least, to fill. This modelizes a complex course pursuit problem because the aspiration gap is adaptive. It changes with past experience (successes and failures to satisfice).

A three stages model of change. Each period, the agent, in the following order, "considers" change and "estimates" if it is worthwhile to change, "implements" a worthwhile change, and finally "exploits this change". Our variational model generalizes the famous two stages "exploration-exploitation" model (March, 1991), adding an intermediate stage where capabilities must be build.

A) In the initial "consideration phase", the agent, i) considers his present situation. He diagnostics the present statu quo (what goes wrong), makes more precise his present aspiration level, considers the gap between them and the frustration feelings this generates, ii) makes more explicit the intention to change instead of to renounce, iii) choosing to try to change, sets intermediate goals (satisficing levels) to try to fill a portion of his current aspiration gap, iv) explores and imagines new actions which can satisfice the intermediate goals, v) explores and imagines capabilities (bundle of resources used in a given order) to be able to do and repeat these new actions several times, vi) estimates the payoffs and utilities related to these new actions, vii) estimates the costs to build new capabilities, including the psychological costs to overcome resistance to change (inertia), for example the costs to break habits. At this stage the agent is able to know if it is worthwhile to change.

B) Then, the agent builds new capabilities. This is an "implementation" or "preparation to change" stage.

C) The last phase of the current period is the "exploitation" stage.

Our consideration and implementation phases A) and B) represent the variational stage. They modelize in much more details the March's "exploration" stage (March,1991). However, in our variational model, the term exploration becomes misleading because consideration and implementation activities include much more than exploration activities, as the list above can show.

Emotions drive the process of change. Contentent and frustration co-evolve. If the agent succeeds to fill a minimal portion of his aspiration gap, he feels some contentment and, at the same time, some deception-frustration to do not have filled the whole aspiration gap. Next period the goal pursuit continues. The agent sets a new aspiration level, which determines a new aspiration gap, ...and so on.... This goal pursuit can be punctuated, entwining a succession of periods where nothing changes (repeating the same temporary routine) with periods of rapid changes. Changes can be incremental (gradual, adaptive) or drastic (revolutionary) and more or less frequent (in our model the length of each period is endogenous).

Our findings. Our concept of variational rationality wants

i) to promote a unified framework for different theories of change where all those approaches can be more easily compared (to save space see our short section 12 on applications to theories of change, for more detailled applications see Soubeyran A, 2010a).

ii) to find conditions such that along a path of worthwhile changes (see our two mains results, section 7 and section 10),

a) motivation to change and resistance to change go to zero, b) the length of the variational stage (where the agent considers and implements change) goes to zero, and the total time spend to change is finite, c) worthwhile to change temporary routines converge in term of the resistance to change quasi metric, d) the agent makes small steps and the length of the worthwhile to change path is finite, e) convergence is in finite time if there are fixed costs to change.

iii) to find conditions such that, if, each step, the agent is able to "satisfice enough" along a path of worthwile changes, there is f) extinction of the aspiration gap**,** g) convergence to an aspiration trap (which exists), making small steps**.** Furthermore**,** h) an aspiration trap (with zero aspiration gap) is also a worthwhile to change trap (the worthwhile to change set shrinks to one point, this trap).

Then, points ii) and iii) give conditions for the existence of a routinization process where, learning less and less, the agent's temporary routine converges to an aspiration trap which is also a cognitive trap (where learning stops), like permanent habits and routines.

Related litterature. Our paper is at the intersection of a huge litterature. To save space let us mention,

1) Behavioral economics, including bounded rationality (satisficing processes, Simon, 1956, 1978), prospect theory (framing processes and variable preferences, Kahneman-Tversky, 1979), evolutionary economics and economics of change (Schumpeter, 1950, Nelson-Winter, 1982)....

2) Psychology: staged models of change (among a long list, Lewin (1947,1951), FFFF), motivation, goal setting, goal striving, goal balancing, emotion, belief, inertia, resistance to change (Lewin, 1947) and learning theories. Self regulation theories, self efficacy, self esteem, FFFF

3) Management : at the individual level and organizational levels, the resource based view (resources and capabilities, routines, see Mahoney, 2005 for a survey), consideration processes (Ranjbarian-Kia, 2010), organizational learning (the "exploration-exploitation" trade off, March, 1991).....

4) Political Sciences: muddling through (Lindblom, 1959), theory of transitions, reforms, institutional change (Williamson, 1991)....

5) Variational analysis in Applied Mathematics and dynamic games: optimization and variational principles (Brezis-Browder, 1976, Ekeland 1974), inexact proximal algorithms (Solodov-Svaiter, 2000), local search optimization (Hoos-Stutzle, 2005, Attouch-Soubeyran, 2010),....game theory (convergence to a Nash equilibrium and "learning to play Nash").

Summary The paper is organized as follows. After the introduction, section 1 gives a simple example of our model. Sections 2, 3, 4, 5 define actions (and capabilities), temporary routines, dynamic capabilities and resistance to change. Section 6 presents our variational model of change, which entwins repetitive and variational phases. Section 7 defines worthwhile changes. Section 8 shows when worthwhile changes converge. Section 9 defines aspiration and consideration traps (permanent routines). Section 10 presents the goal pursuit model. Section 11 gives conditions for disparition of the aspirations gap, and convergence to an aspiration trap when the agent uses worthwhile changes and "satisfices enough". Section 12 considers the case of "no regret", including consideration costs ex post. Section 13 examines very briefly several applications to the theories of change. Conclusion, references and appendix follow.

2 The simplest model of "the unsatisfied man"

In this simple example we identify actions and temporary routines, an action $x \in X$ being repeated only one time ($t(x) = 1$), unless the agent reachs a behavioral trap.

The dynamics of aspiration gaps Let $F(x_0) = [x_0, +\infty[\subset X = \mathbb{R}_+$, be the "ability set" of an agent, the subset of present actions he is able to do. More precisely consider a sportman and the subset of non negative efforts $F(x_0)$ that he can perform. In this simple example we identify actions, efforts $x \in F(x_0)$, and the capabilities to do them, $\chi(x) \in \boldsymbol{\chi}$.Then, in this example, we identify the "ability set" of actions $F(x_0)$ and the "capability set"of means (capabilities to do actions). Furthermore, the ability set does not change from step to step (no learning): $F(y) = F(x_0)$ for all $y \in X$.

Let $g(x) \in \mathbb{R}$ be the performance of the agent when he does the effort level $x \in F(x_0)$ and $g(.) : x \in F(x_0) \longmapsto g(x) \in \mathbb{R}$ be his performance function, bounded above, and strictly increasing (the more effort, the higher the performance $g(x)$).Let $\overline{g} = \sup \{g(x), x \in F(x_0)\} < +\infty$ be the supremum performance level. For simplification consider the utility of the performance $g(x)$ as $U_+ [g(x)] = g(x)$.Take (as example 1) the function $g(x) = x/(1 + x)$ where $\overline{g} = 1$.Let $\widetilde{g} \in \mathbb{R}$ be a given and satisficing performance and utility level (a goal) such that the agent is happy if he can do a performance higher or equal than this level. This satisficing level is reachable if the agent can find an effort $x \in F(x_0)$ such that $g(x) \geq \widetilde{g}$. In this case the agent satisfices (Simon, 1955). By definition the supremum is the highest reachable satisficing level. For any $\widetilde{g} > \overline{g}$, there exists no effort level $x \in F(x_0)$ such that $g(x) \geq \widetilde{g}$, while for any $\widetilde{g} < \overline{g}$ it exists $x \in F(x_0)$ such that $g(x) \geq \widetilde{g}$. Furthermore $x^* \in F(x_0)$ is a maximum effort iff $g(x^*) = \overline{g}$. This is the case for $x^* = \mu/2$ if the performance function is $g(x) = \mu x - x^2$, with $\mu > 0$ (example 2). Then, the supremum represents the aspiration level of the agent. The reader must notice that in our variational model the aspiration level is different from a satisficing level. Simon (1955), and all his followers use the terminology "aspiration level" for what is for us a "satisficing level". He does not considers aspiration levels as we do. In the first example the aspiration level cannot be reached, and it can be in the second example.

First step. If the agent chooses to perform the initial effort x, his performance and utility levels are $U_+ [g(x)] = g(x)$. As in the traditional utility theory the agent feels happy of the utility of his performance and $U_+ [g(x)]$ represents a contentment feeling. We add a negative feeling, the agent being unhappy of not having reached his aspiration level, feeling the frustration $U_- [\overline{g} - g(x)] = v(\overline{g} - g(x)), 0 < v < 1$. If the agent starts from the statu quo level of effort x the present difference $\overline{a}(x) = \overline{g} - g(x) \geq 0$ represents his aspiration gap. Then, the net utility of the agent is $U_+ [g(x)] - U_- [\overline{a}(x)]$. In general the agent does not know his true aspiration level $\overline{g}$ and sets an estimation $\widehat{g}(x) \lesseqgtr \overline{g}$ which represents his perceived aspiration level. Then, his perceived aspiration gap at x is $\widehat{a}(x) = \widehat{g}(x) - g(x) \geq 0$.The agent sur estimates (perfectly estimates) or underestimates his true aspiration level $\overline{g}$ if $\widehat{g}(x) > \overline{g}, \widehat{g}(x) = \overline{g}$, or $\widehat{g}(x) < \overline{g}$ (the agent having a too high, perfect or too low self esteem). If, in this first step, the aspiration gap of the agent is zero, $\widehat{a}(x) = \widehat{g}(x) - g(x) = 0$, this means that he has done immediately an effort which gives him no further motivation to change it. The equality $\widehat{g}(x) = g(x)$ defines an aspiration trap $x^* = x$.When, by luck, the agent perfectly estimates his aspiration level, $\widehat{g}(x) = \overline{g}$, the aspiration trap becomes an optimal effort such that $g(x) = \overline{g}$ and the agent optimizes.

Second step. If, in the first step, the effort x is not an aspiration trap, the agent feels the aspiration gap $\widehat{a}(x) = \widehat{g}(x) - g(x) > 0$.Having done effort x in the first step, and unless he is depressed, a pro-active agent will not renounce to partially fill his aspiration gap. He will choose to transform his initial frustration feeling into the motivation to try reduce it. He tries to make a higher effort $y > x$ and hence a higher performance level $g(y) > g(x)$ to lower his frustration feeling from $\widehat{a}(x) = \widehat{g}(x) - g(x) > 0$ to $0 \leq \widehat{g}(x) - g(y) < \widehat{a}(x)$.

Third step. If the effort y is an aspiration trap, the process stops. Suppose not. Then, $\widehat{g}(x) - g(y) \geq 0$ is the residual aspiration gap of the agent which can set a new aspiration level $\widehat{g}(y) \lesseqgtr \widehat{g}(x)$. His new aspiration gap becomes $\widehat{a}(y) = \widehat{g}(y) - g(y) \geq 0$.The adaptive dynamic of both the aspiration level and the aspiration gap continues.....etc....

Improving To simplify, suppose that the agent perfectly estimates aspiration level $\widehat{g}(x) = \overline{g}$ and does not adapt step by step this aspiration level: $\widehat{g}(x) = \overline{g}$ for all $x \in X$. Then, if the agent succeeds to improve step by step, $g(y) > g(x)$ he will reduce his aspiration gap, $0 \leq \widehat{a}(y) = \overline{g} - g(y) < \widehat{a}(x) = \overline{g} - g(x)$. The succession of inequalities $\overline{g} \geq g(x_{n+1}) > g(x_n)$ for all $n \in N$ implies that the sequences of performances and aspiration gaps converge, $g(x_n) \longrightarrow g^*, n \longrightarrow +\infty$, $\widehat{a}(x_n) \longrightarrow \widehat{a}^* \geq 0, n \longrightarrow +\infty$. This does not imply tha the aspiration gap vansihs. The frustration feelings of the agent does not disappear in the long run. The agent must "improve enough" to succeed to do that.

Satisficing and "satisficing enough" Suppose, for simplification, that the agent perfectly estimates his true aspiration level $\overline{g}$,ie $\widehat{g}(x) = \overline{g}$ for all $x \in F(x_0)$ and, as before, does not adapt step by step his aspiration level $\widehat{g}$. Suppose that, starting from the effort level x, and the strictly positive aspiration gap $\widehat{a}(x) = \overline{g} - g(x) > 0$, the agent sets the satisficing level $\widetilde{g}(x) \in \left]g(x), \overline{g}\right[$, ie $\widetilde{g}(x) = g(x) + \varepsilon(x), 0 < \varepsilon(x) < \widehat{a}(x)$ where $\varepsilon(x) > 0$ represents the satisficing gap at x.Then, the agent satisfices when he is able to find $y \in F(x_0)$ such that $g(y) \geq \widetilde{g}(x) \Longleftrightarrow g(y) - g(x) \geq \varepsilon(x) > 0$.This last inequality means that he is able to "improves enough", more than $\varepsilon(x) > 0$.The satisficing gap $\varepsilon(x) = \theta(x)\left[\overline{g} - g(x)\right] > 0$ fills a portion $0 < \theta \leq \theta(x) < 1$ of the aspiration gap $\widehat{a}(x)$. The agent "satisfices enough" if he is able to find an action $y \in F(x_0)$, $g(y) \geq \widetilde{g}(x)$, which reachs a satisficing level $\widetilde{g}(x)$ such that the satisficing gap fills a high enough portion of the aspiration gap, $0 < \theta \leq \theta(x) < 1$. In this case $\varepsilon(x) = \theta(x)\left[\overline{g} - g(x)\right] \geq \theta\left[\overline{g} - g(x)\right] > 0$ and $g(y) - g(x) \geq \theta\left[\overline{g} - g(x)\right] > 0$.

Suppose, for simplification, that the agent estimates perfectly his aspiration level, ie $\widehat{g} = \overline{g} < +\infty$. Let us show that when the agent is able to "satisfice enough" each step, his aspiration gap vanishs. This is not the case when he is only able to satisfice each step.

Proof: suppose that, each step, $g(x_{n+1}) - g(x_n) \geq \theta\left[\overline{g} - g(x_n)\right] > 0, n \in N$.Then, the sequence of performances $\{g(x_n)\}$ is increasing and bounded above by the supremum $\overline{g} < +\infty$. Hence this sequence converges: $g(x_n) \longrightarrow \overline{g}, n \longrightarrow +\infty$ and the marginal performance goes to zero, $g(x_{n+1}) - g(x_n) \longrightarrow 0, n \longrightarrow +\infty$. This implies that the aspiration gap $\widehat{a}(x_n) = \overline{g} - g(x_n) \geq 0$ vanishs along the "satisficing enough" process: $\widehat{a}(x_n) \longrightarrow 0, n \longrightarrow +\infty$.

Optimization as a limit case: an aspiration gap approach of the Weierstrass theorem Let us show that a maximum of a performance function can be the end point of a "satisficing enough process".

Definition (Chen- Cho- Yang, 2002). Let (X, d) be a metric space. The function $g(.) : x \in X \longmapsto g(x) \in \mathbb{R}$ is upper semicontinuous from below iff for any sequence $\{y_k\}$ such that, i) $g(y_{k+1}) \geq g(y_k), n \in N$, ii) $\lim_{k \longrightarrow +\infty} g(y_k) = g^* < +\infty$ and iii) $\lim_{k \longrightarrow +\infty} y_k = x^*$, then, $\lim_{k \longrightarrow +\infty} g(y_k) = g^* \leq g(x^*)$ (the increasing sequence of performances $g(y_k)$ continues to increase when the action pass from y_k to x^*). This hypothesis is weaker than the upper semicontinuous hypothesis.

Weierstrass theorem. Let (X, d) be a compact metric space and $g(.) : x \in X \longmapsto g(x) \in \mathbb{R}$ be a bounded above and upper semicontinuous function from below. Then, starting from any initial point $x_0 \in X$, it exists a "satisficing enough" path which converges to a maximum of $g(.)$.

Proof: using the argument given just above, and starting from any initial point $x_0 \in X$, we can build a "satisficing enough" process $\{x_n\}$ such that $\{g(x_n)\}$ is not decreasing and $g(x_n) \longrightarrow \overline{g}, n \longrightarrow +\infty$.Then, X being compact, we can extract a converging subsequence $\{y_k = x_{n_k}\}$, where $y_k \longrightarrow x^*, n \longrightarrow +\infty$, $\{g(y_k)\}$ is not decreasing and $\lim_{k \longrightarrow +\infty} g(y_k) = \overline{g}$. If $g(.)$ is upper semicontiuous from below, this implies

$\overline{g} = \lim_{k \longrightarrow +\infty} g(y_k) \leq g(x^*) \leq \overline{g}$. Then, $g(x^*) = \overline{g}$.

Comment: the agent reachs a maximum of the function $g(.)$, which is a behavioral trap in this very specific context.

The variational man: the course pursuit between aspiration gaps and capabilities In this simple setting, our mathematical problem is, each step, to solve approximatively (giving only a temporary satisficing solution), the course pursuit problem $\overline{g}(x_n) = \sup \{g(y), y \in F(x_n)\}, n \in N$ where the ability set $F(x_n) \subset X$ changes.The rest of our paper elaborates on this simple model. It considers temporary routines $r = (x, t) \in \mathbf{R}$ where an action x_n is repeated t times. The ability set can change with experience $e_n \in \mathbf{E}$ ans well as the performance function $g(., e_n)$. The course pursuit model co-determines, each step, means (the ability set and the capability set) and ends (aspiration levels and satisficing levels). It modelizes the formation, each step, of a consideration set. It adds costs to change and exploration costs (among them search costs to satisfice). It shows how satisficing drives the exploration process, and how the agent can make wrong estimations of his true aspiration level. No compacity assumptions are made.

3 Actions, and capabilities

An action $x \in \mathbf{X}$ is both an outcome and a process (a succession of operations). An agent produces something, the outcome $q = q(x) \in \mathbf{Q}$, using means (resources and ingredients and the ways and abilities to use resources to transform and combine ingredients). He follows a transformation process $\sigma(x) \in \mathbf{\Sigma}$ to produce this outcome q, starting from an initial state q_{-1} (see Soubeyran A, 2010a). The initial state q_{-1} is a list of resources and ingredients that the agent can use (he has a physical access to them, he has the right to use them, he knows how to use them, and how to combine ingredients).

The capability $\chi(x) = (h(x), w(x), \epsilon(x)) \in \boldsymbol{\chi}$ to do an action x one time requires,

i) to have, available for use ("to have access to"), a stock of resources and ingredients $h(x) \in \mathbf{H}$ (to own, to buy, to build, to learn,....), and to delete resources and ingredients which represent obstacles (to sell, to eliminate, to forget, to break habits,...),

ii) to know the ways $w(x) \in \mathbf{W}$ of using these resources and ingredients.They represent the ways to activate these resources and the ways to combine and transform ingredients to be able to produce a final outcome,

iii) to have the abilities $\epsilon(x) \in \Xi$ to perform these ways of doing. The capability to repeat an action x several times ($t(x)$ times) includes maintenance activities to re-generate the resources which have been used several times.

4 A repetitive stage : a temporary routine

A temporary routine is an action which is repeated several times, the action being itself a succession of operations. It represents a temporary habit at the indivual level and a temporary routine at the organizational level. We will use the term temporary routine to describe both of them. Formally the temporary routine $r = r(x) = (x, t(x)) \in \mathbf{R}$ repeats $t(x) \in \mathbf{N}$ times an action $x \in \mathbf{X}$. The outcome of this temporary routine is $Q(r) \in \mathbf{Q}$ and the outcome per action is $q(r) = Q(r)/t(x) \in \mathbf{Q}$. The time scale is the following. A unit of time is an hour. The agent spends $T(r) \in \mathbb{R}_+$ hours to do this temporary routine. The agent does action x only one time each day. Then, the temporary routine $r = (x, t(x))$ runs over $N(r) = t(x)$ days and the mean time spend, each day, to do action x only one time is $\delta(r) = T(r)/N(r)$, this action being repeated $t(x)$ times (this formulation can modelize learning by doing).We will suppose that this mean time is lower than the length $\overline{\delta} > 0$ of the day, $0 < \delta(r) \leq \overline{\delta}$.

Let $\chi(r) \in \boldsymbol{\chi}$ be the capability to repeat action x several times ($t(x)$ times). It represents the repeated ability to use means, (to have access to them, to know the ways to use them, and to be able to use them, having access to the means to use the means...) to finally produce a final outcome several times.

Define a capability map $\chi(.) : r \in \mathbf{R} \longrightarrow \chi(r) \in \boldsymbol{\chi}_r$ where $\boldsymbol{\chi}_r \subset \boldsymbol{\chi}$ represents the subset of all capabilities to do the temporary routine r. A capability map chooses one of them, $\chi(r) \in \boldsymbol{\chi}_r$, for each temporary routine $r \in \mathbf{R}$.

The benefit coming from the outcome $Q(r)$ is $B(Q(r)) \in \mathbf{B} = \mathbb{R}_+$.

The cost spend to perform this temporary routine is

$K(r) = K_P(r, \delta(r)) + K_M(\chi(x), \chi(x), t(x)) \in \mathbf{K} = \mathbb{R}_+$.This cost includes two kinds of costs,

i) production costs $K_P(r, \delta(r)) \geq 0$ are the costs to perform action x several times ($t(x)$ times). They depend of the action x to be done, the number of repetitions $t(x)$, and of the mean time $\delta(r)$ spend to do action x one time. They represent the costs to use resources to combine and transform ingredients to finally produce a final outcome $t(x)$ times. Production costs incorporate "learning by doing" aspects when mean production costs $K_P(r, \delta(r))/t(x)$ decrease with the number of repetitions $t(x)$. Production costs also incorporate reactivity aspects when they depend of the mean time $\delta(r)$ spend to do action x one time (the speed of doing). With no learning by doing and no reactivity aspects, these "costs to do" are $K_P(r) = t(x)k(x)$ where $k(x)$ is the usual cost to do action x one time. In general $k = k(r, \delta(r))$.

ii) costs to be able to repeat an action several times, like maintenance and regeneration costs $K_M(\chi(x), \chi(x), t(x)) \geq 0$. They represent costs to be able to pass from the capability $\chi(x)$ to do action x to the capability $\chi(x)$ to do action x, and this, $t(x)$ times). Spending these costs allow to regenerate the used portions of resources and ingredients and to do not forget how to use resources and how to combine ingredients. To pass from having done an action x to be able to do this action again and again, $t(x)$ times do not include new operations from one time to the next.

The payoff of the temporary rountine is $G(r) = \Pi\left[r, Q(r)\right] = B(Q(r)) - K(r)$.

The net per action payoff is $g(r) = G(r)/t(x)$ for $t(x) > 0$. If the action x is repeated one time, then the temporary routine $r(x) = (x, t(x) = 1)$ represents action x, and the net per action payoff is $g(x) = \Pi[x, Q(x)]$. Notice that this formulation emphasizes the chain " action $x \in X \curvearrowright$ outcome $Q(x) \curvearrowright$ payoff $\Pi[x, Q(x)] = G(x)$"

which is one of the characteristic of the expectance-valence approach of motivation (Vroom, 1964) (here probabilities are not introduced for simplification, see however section 8). When experience $e \in E$ matters, the payoff of doing action x one time is $G(r, e) = t(x)g(r, e)$.

5 A variational stage : dynamic capabilities

Let $\omega \in \Omega(\chi(r), \chi(s))$ be the (dynamic) capability to change, ie to pass from the capability $\chi(r)$ to perform the temporary routine $r = r(x) \in \mathbf{R}$ to the capability $\chi(s)$ to perform the temporary routine $s = r(y) \in \mathbf{R}$. Let $T(\omega)$ be the time spend to do that. The symbol ω represents a way of change, including a path of change and the way to follow this path. If, starting from the temporary routine $r = r(x) = (x, t(x))$, the agent wants to repeat again action x several times (eventually a different number of times, $t'(x)$ times), then, the new temporary routine is $r'x) = (x, t'(x))$. In this case $\omega \in \Omega(\chi(r), \chi(r'))$ represents the (dynamic) capability to stay, ie to pass from the capability $\chi(r)$ to the capability $\chi(r')$ and $T(\omega)$ is the time spend to be able to repeat again action x several times, eventually a different number of times if $t'(x) \neq t(x)$. Dynamic capabilities can be composed: if $\omega_1 \in \Omega(\chi(r_1), \chi(r_2))$ and $\omega_2 \in \Omega(\chi(r_2), \chi(r_3))$, then $\omega_1 o \omega_2 \in \Omega(\chi(r_1), \chi(r_3))$ is a dynamic capability to pass from the capability $\chi(r_1)$ to the capability $\chi(r_3)$ which takes the time $T(\omega_1) + T(\omega_2)$.Teece and alii (1997) have been the first to emphasize the importance of the concept of "dynamic capabilities" (without any formalization). Several authors have contested the usefulness of the concept. Our formalized approach clearly shows the opposite.

6 Resistance to change: the desutility of the costs of building capabilities

Costs to be able to change "Costs to do" an action one time or several times, ie the cost to do a temporary routine $s = (y, t(y)) \in \mathbf{R}$ are not the same as the "costs to be able to do" it. Formally, the costs to do an action y several time ($t(y)$ times) are $K(s) \in \mathbb{R}_+$. They are very different from the costs $C[\chi(r), \chi(s), \omega, T(\omega)]$ to be able to do a new temporary routine $s \in \mathbf{R}$, using a way to change $\omega \in \Omega(\chi(r), \chi(s))$ from the temporary routine $r = (x, t(x))$ to $s = (y, t(y))$. The term $C[\chi(r), \chi(s), \omega, T(\omega)]$ defines an ex ante cost to be able to change, a preparation cost, to be spend before being able to do an action (see Soubeyran A, 2010a). A way of change $\omega = (p, l(p))$ represents a physical or cognitive path of change $p \in P(\chi(r), \chi(s))$ of length $l(p) \geq 0$. The time spend to follow it is $T(\omega) \geq 0$. The speed of change along this path is $v = l(p)/T(\omega)$. The agent builds new capabilities, starting from old ones. He deletes old resources (sells, forgets, throws), acquires and gathers new resources (buys, builds, learns), conserves others (maintain them available for use). He gets the ability to have access to these resources, he learns how to use them to combine and transform ingredients, following some protocole (some way of doing). Costs to change modelize learning costs, inertia, ..., consideration costs, costs to set goals, exploration and evaluation costs ....(see Soubeyran A, 2010a), costs to break temporary habits and routines.

Costs to be able to stay: maintenance costs To make the parallel with costs "to be able to move", $C\left[\chi(r), \chi(s), \omega, T(\omega)\right], \omega \in \Omega(\chi(r), \chi(s)$ where $r = (x, t(x))$ and $s = (y, t(y))$ are two different temporary routines which are the repetitions of two different actions x and $y \neq x$, costs "to be able to stay" can be modelized as $C\left[\chi(r), \chi(r'), \varsigma, T(\varsigma)\right], \varsigma \in \Omega(\chi(r), \chi(r'))$, where the new temporart routine $r' = r'(x) = (x, t'(x)) \in \mathbf{R}$ repeats the same action x a number of times $t'(x)$ (the same or different from $t(x)$). But it is more convenient to modelize these costs "to be able to stay" as maintenance (or regeneration) costs $C\left[\chi(r), \chi(r'), \varsigma, T(\varsigma)\right] = K_M(\chi(x), \chi(x), t(x)) \geq 0$ and to include them, as done before (see section1), in the payoff $G(r) = B(Q(r)) - K(r)$ where $K(r) = K_P(r, \delta(r)) + K_M(\chi(x), \chi(x), t(x))$. If, the previous period, the agent has been able to do an action x several times ($t(x)$ times), his costs to be able to do again the same action x several times ($t'(x)$ times) are the costs to gather new stocks of the same resources and ingredients to regenerate the portion of used resources. Knowledge costs (to know which resources to use, which ingredients to combine using these resources, how to find these resources and ingredients, how to use resources and how to combine ingredients, remembering the protocole of all the operations) decrease because of learning by doing. Such knowledge costs necessary to be able to stay (to repeat an action) are included in $K_M(\chi(x), \chi(x), t(x))$.

Intrinsic benefits to change Consider the variational stage where the agent chooses a way of change $\omega \in \Omega(\chi(r), \chi(s))$ of length $\Lambda(\omega) \geq 0$, which allows him to pass from the capability $\chi(r)$ to the capability $\chi(s)$, being able to perform the new routine $s = (y, t(y))$ instead of the old routine $r = (x, t(x))$. During this stage of change, depending of the strenght of his intrinsic motivation, he gets the gross intrinsec benefit to change $B^{\$}\left[\chi(s), \chi(s), \omega, T(\omega)\right] \geq 0$. This means that the agent can (or not) enjoy to discover, and to start new challenges. He can loves these periods where he researchs, creates, innovates, learns.

Infimum costs to be able to change Consider the subset of temporary routines $r = r(x) = (x, t(x)) \in \mathbf{R}$ with the same given number of repetitions $t(x) = t \geq 1$ for all $x \in \mathbf{X}$. Let $\mathbf{R}_t = \{r = r(x) = (x, t(x)) \in \mathbf{R}, t(x) = t\} \subset \mathbf{R}$ be such a subset.

Define a capability map $\chi(.) : r \in \mathbf{R}_t \longrightarrow \chi(r) \in \boldsymbol{\chi}_r$ where $\boldsymbol{\chi}_r \subset \boldsymbol{\chi}$ represents the subset of all capabilities to do the temporary routine r. The capability map chooses one of them, $\chi(r) \in \boldsymbol{\chi}_r$. Remember that, i) a temporary routine r is an action repeated several times, the action being itself a succession of operations, ii) a capabilty represents the ability to use means, (to have access to them, to know the ways to use them, and to be able to use them, having access to the means to use the means,....to finally produce a final outcome).

Hypothesis 1: if two temporary routines are different, then, capabilities to do these two temporary routines are different. Formally, if $r = (x, t), s = (y, t) \in \mathbf{R}_t, s \neq r \Longrightarrow \chi(s) \neq \chi(r)$. This is equivalent to $\chi(s) = \chi(r) \Longrightarrow s = r$ ie $y = x$.

Definition.

Let $\underline{C}(\chi(r), \chi(s)) = \inf \{C[\chi(r), \chi(s), \omega, T(\omega)], \omega \in \Omega(\chi(r), \chi(s))\} \geq 0$

be the infimum costs to be able to pass from the capability to do the temporary routine $r = (x, t) \in \mathbf{R}_t$ to the capability to do the temporary routine $s = (y, t) \in \mathbf{R}_t$.If $s = r$, costs to be able to change are zero (by definition): $C[\chi(r), \chi(r), \varsigma, T(\varsigma)] = 0$, for all $\varsigma \in \Omega(\chi(r), \chi(r))$ because the agent starts no new operation and only repeat old ones.

Hypothesis 2: when capabilities to do temporary routines are different infimum costs to be able to change are strictly positive:

$\chi(s) \neq \chi(r) \Longrightarrow \underline{C}(\chi(r), \chi(s)) > 0$.Then,

$\underline{C}(\chi(r), \chi(s)) = 0 \Longrightarrow \chi(s) = \chi(r) \Longrightarrow s = (y, t) = r = (x, t) \Longleftrightarrow y = x$.

Quasi distance between temporary routines. The infimum cost

$\underline{C}(\chi(r), \chi(s)) \geq 0$ to be able to change from a temporary routine $r = (x, t) \in \mathbf{R}_t$ to a new temporary routine, $s = (y, t) \in \mathbf{R}_t$ defines a quasi distance between capabilities to perform temporary routines with the same number of repetitions.

Consider the succession of two ways of change $\omega_1 o \omega_2 \in \Omega(\chi(r_1), \chi(r_3))$ where $\omega_1 \in \Omega(\chi(r_1), \chi(r_2))$ and $\omega_2 \in \Omega(\chi(r_2), \chi(r_3))$. Define the costs to be able to change over the succession of these two ways of change as the sum of the costs to be able to change over each way of change, $C[\chi(r_1), \chi(r_3), \omega_1 o \omega_2, T(\omega_1) + T(\omega_2)] =$

$C[\chi(r_1), \chi(r_2), \omega_1, T(\omega_1)] + C[\chi(r_2), \chi(r_3), \omega_2, T(\omega_2)]$.

Then,

i) $\underline{C}(\chi(r), \chi(r)) = 0$, for all $\chi(r) \in \boldsymbol{\chi}$.

ii) $\underline{C}(\chi(r), \chi(s)) = 0 \Longrightarrow \chi(s) = \chi(r)$ (see before) and

iii) $\underline{C}(\chi(r_1), \chi(r_3)) \leq \underline{C}(\chi(r_1), \chi(r_2)) + \underline{C}(\chi(r_2), \chi(r_3))$ for all $r_1, r_2, r_3 \in \mathbf{R}$. (see Mennucci, 2007 for a survey on quasi metrics).

Let us consider now the triangular inequality. Given the temporary routines $r_1, r_2, r_3 \in \mathbf{R}$, the related capabilities to do them $\chi(r_1), \chi(r_2), \chi(r_3) \in \boldsymbol{\chi}$,and the ways of change $\omega_1 \in \Omega(\chi(r_1), \chi(r_2))$ and $\omega_2 \in \Omega(\chi(r_2), \chi(r_3))$, we have,

$\underline{C}(\chi(r_1), \chi(r_3)) \leq C[\chi(r_1), \chi(r_2), \omega_1 o \omega_2, T(\omega_1) + T(\omega_2)] =$

$C[\chi(r_1), \chi(r_2), \omega_1, T(\omega_1)] + C[\chi(r_2), \chi(r_3), \omega_2, T(\omega_2)]$ which implies

$\underline{C}(\chi(r_1), \chi(r_3)) - C[\chi(r_1), \chi(r_2), \omega_1, T(\omega_1)] \leq C[\chi(r_2), \chi(r_3), \omega_2, T(\omega_2)]$

for a given $\omega_1 \in \Omega(\chi(r_1), \chi(r_2))$ and for any $\omega_2 \in \Omega(\chi(r_2), \chi(r_3))$.

Then, $\underline{C}(\chi(r_1), \chi(r_3)) - R[\chi(r_1), \chi(r_2), \omega_1, T(\omega_1)] \leq \underline{C}(\chi(r_2), \chi(r_3))$. This imples that

$\underline{C}(\chi(r_1), \chi(r_3)) - \underline{C}(\chi(r_2), \chi(r_3)) \leq R[\chi(r_1), \chi(r_2), \omega_1, T(\omega_1)]$ for all $\omega_1 \in \Omega(\chi(r_1), \chi(r_2))$.

Then, $\underline{C}(\chi(r_1), \chi(r_3)) - \underline{C}(\chi(r_2), \chi(r_3)) \leq \underline{C}(\chi(r_1), \chi(r_2)$ for all $r_1, r_2, r_3 \in \mathbf{R}$.

The result follows.

Resistance to change **Definition**: resistance to change is the desutility of the net costs to change. Formally

$R[\chi(r), \chi(s), \omega, T(\omega)] = D[C[\chi(r), \chi(s), \omega, T(\omega)]]$, where $D(C) \geq 0$ is the desutility of the cost to change $C \geq 0$.We will suppose $D(.)$ zero at zero, and strictly increasing. At the group level the concept of resistance to change comes from Lewin (1951), yet with no formalization (in Management Sciences, see Rumelt,1995).

The distance between two temporary routines $r = (x, t(x))$ and $s = (y, t(y))$ is $d(r, s) = d_X(x, y) + |t(y) - t(x)|$ where $d_X(x, y) \geq 0$ is a distance between the two actions x and y. When temporary routines repeat the same number of times an action, ie $r, s \in \mathbf{R}_t$, then, $d(r, s) = d_X(x, y)$.Infimum costs to change are high if $\underline{C}(\chi(r), \chi(s)) \geq d(r, s)$ for all $r, s \in \mathbf{R}_t$.

7 A variational model of change: entwinning repetitive and variational phases

A variational model of change Consider one period, where, the period before, the agent has done the temporary routine $r(x) = (x, t(x)) \in \mathbf{R}$, repeating $t(x)$ times an action $x \in X$. Let $g(r(x), e) \in \mathbb{R}$ be his "per action payoff" or "per action utility" to repeat action $x \in X$ this period, given his own experience $e \in E$. Within this period the agent passes from doing the temporary routine r next period to do the temporary routine $s = (y, t(y))$ at the end of this period, after some preparation stage.

The time scale is the following. Divide the present period in three consecutive stages $(c), (\omega)$ and (s) of lengths $T(c), T(\omega)$ and $T(s)$ units of time (say hours): first a "consideration stage" (c) where the agent sets goals, imagines and estimates change, then, a "change stage" (ω) where the agent implements the chosen change from the previous temporary routine r done last period to a new temporary routine $s = (y, t(y))$ to be perfomed in the repetitive and last stage (s) where the agent exploits change. The two first stages $v = (c) \cup (\omega)$ represent the variational stage v of length $T(v) = T(c) + T(\omega)$. This is a preparation stage, before being able to perform a new action y several times, $(t(y))$ times) at the beginning of the repetitive stage (s).The length of the period is $T(c) + T(\omega)+ T(s)$ hours. If we divide the period in days, where the length of each day is $\overline{\delta} > 0$,the consideration stage and the change stage lasts $N(c)$ and $N(\omega)$ days, where the time spend per day to consider and change are

$0 \leq \delta(c) = T(c)/N(c) \leq \overline{\delta}$ and $0 \leq \delta(\omega) = T(\omega)/N(\omega) \leq \overline{\delta}$. In the repetitive stage (s) we will suppose that the agent carries out only one action y each day. Then, this exploitation stage (s) lasts $t(y) = N(s)$ days and the mean time spend to do one time action y is $0 < \delta(s) = T(s)/N(s) \leq \overline{\delta}$, this action being repeated $t(y)$ times. Each day, the difference $\overline{\delta} - \delta(z) \geq 0$ represents some free time after having done task z. This diachronic model of change generalizes the famous "exploration-exploitation" model of March (1991) where the variational stage includes exploration (search) as a particular activity falling within the consideration stage. For a synchronic version where, each day, the agent spends some times to exploit and the remaining time to variational activities to prepare change next period, see Soubeyran A (2010a). In the present model the three consecutive stages are,

1) first **a consideration stage** (c) which lasts $\Lambda(c) > 0$ units of time (hours). This stage is a succession of several sub-stages:

-first the agent, dealing with emotions, feels that something is wrong, that a need must be filled. This pushs him, i) to make a diagnostic of the present situation which helps him to evaluate his "per action payoff" $g(r(x), e) \in \mathbb{R}$, repeating $t(x)$ times action $x \in X$ this period, ii) to materialize his frustration feeling, setting a per action aspiration level $\widehat{g}(r(x), e) \geq g(r(x), e)$, where the frustration gap $\widehat{g}(r(x), e) - g(r(x), e) \geq 0$ generates a frustration feeling if it is strictly positive (for the formation of aspiration levels, see Soubeyran A, 2010b).

- then, either the agent renounces to fill a portion of his frustration gap (depression, amotivation), or he transforms this frustration feeling into an aspiration gap $\widehat{a}(r(x), e) = \widehat{g}(r(x), e) - g(r(x), e) \geq 0$.

In the second case where he has the motivation to fill a portion of his aspiration gap,

-he builds, more or less seriously, a consideration set $F(r(x), e) \subset \mathbf{R}$ of temporary routines $r(y) \in \mathbf{R}$, hopping that some of these temporary routines can allow him to improve and fill a portion of his aspiration gap. If these new temporary routines do not exist the agent must imagine them.

-he explores and evaluates much more seriously temporary routines within the consideration set, $r(y) \in F(r(x), e)$, their related "per action payoff" $g(r(y), e)$, and the capabilities $\chi(r(y)) \in \boldsymbol{\chi}$ required to be able to perform these new temporary routines.

-he sets a satisficing level $\widetilde{g}(r(x), e) \in \left]g(r(x), e), \widehat{g}(r(x), e)\right[$, strictly higher than the "per action payoff" $g(r(x), e)$ and strictly lower than the aspiration level.

-he takes a decision, choosing to move to some new temporary routine $r(y)$ or to stay at $r(x)$, balancing motivation and resistance to change, and dealing with emotions once again.

All these "consideration activities" generate consideration costs which are sunk costs. They only help the agent, at the end of the consideration stage, to be able to choose to stay or to move (they give him the opportunity to be bounded rational). The consideration stage allows the agent to bracket his decision (an important aspect of bounded rationality). He becomes able to balance, within his consideration set, advantages to change and costs to change and to estimate the utility of advantages to change, his motivation to change, and the desutility of costs to change, his resistance to change. All these aspects will be modelized later because we concentrate first on the two last phases of the period, (variation and exploitation), working backward for the ease of exposition.

2) then, **a change stage** starts, as soon as the agent has chosen to move from doing the temporary routine $r = r(x)$ last period to perform a new temporary routine $s = r(y) = (y, t(y))$ this period. The opposite case where, by comparison, the agent prefers to stay (to repeat $t(y)$ times action x, doing the temporary routine $r' = r(x)' = (x, t(y)))$ than to move to s is examined in the next section. This variational stage lasts $T = T(\omega)$ times where the agent builds new capabilities $\chi(r(y)) \in \boldsymbol{\chi}$ to be able to pass from having performed the temporary routine $r(x)$ last period, to be able to perform later, in the exploitation stage of this period, the new temporary routine $r(y) = (y, t(y))$, taking the mean time $\delta(s) > 0$ to do each repeated action y. The cost to build the capability $\chi(r(y))$, starting from having the capability $\chi(r(x))$, following a way of change $\omega \in \Omega(\chi(r(x)), \chi(r(y)))$, which takes the time $T(\omega) \geq 0$ is $C\left[(\chi(r(x)), \chi(r(y)), \omega, T(\omega)\right] \geq 0$. Following this way of change can make the agent happy or unhappy, depending of his personality. If he has some intrinsic motivation to change (enjoys innovation, is happy to change, to learn and to do not repeat the same action every time), his intrinsic benefit to change is, during this variational phase, $B^{\$}\left[(\chi(r(x)), \chi(r(y)), \omega, T(\omega)\right] \geq 0$.

3) finally, **an exploitation stage** which lasts $T(s)$ times where the agent carries out $t(y)$ times action y, getting the exploitation payoff $G(r(y), e) = t(y)g(r(y), e)$.

Remark: the litterature on staged models of change is given in the last section.

8 Worthwhile changes: a prospect approach

In this section we define motivation to change and balance, each step, motivation to change (the utility of advantages to change) and resistance to change (the des-utility of costs to change). Following prospect theory (Kahneman-Tversky, 1979), which is a theory of change, we define the utility of changes, ie the utility of variations with respect to a reference point, rather than the utility of absolute magnitudes ("carriers of value are changes in wealth or welfare, rather than final states"). We choose as an endogenous reference point the present temporary routine (the current statu quo). Then, we balance the utility of gains (for us, advantages to change) over the desutility of losses (for us, net costs to change), adding a lot of psychological and cognitive elements. For simplification, we will consider, from now to the end of the paper, the subset of temporary routines $r = r(x) = (x, t(x)) \in \mathbf{R}_t$ which repeat an action x the same (given) number of times, $t(x) = t \geq 1$ for all $x \in \mathbf{X}$.This does not limit our analysis because several number $n \in \mathbf{N}$ of repetitions of a same temporary routine $r = (x, t)$ modelizes temporary routine $s = (x, nt)$ with a variable number of repetitions nt of the same action x, allowing for a punctuated dynamic (long periods where nothing change and small periods of abrupt changes). The general case works as well, adding unecessary notations.

"Should I stay, should I go" ? Suppose that, within a given period, the consideration stage is finished. Then, at this point of time, the agent is able to choose between "to stay" or "to move",

a) if the agent chooses to move from $r = (x, t) \in \mathbf{R}_t$ to $s = (y, t) \in \mathbf{R}_t$, $y \neq x, t \geq 1$, he enters in the variatonal stage followed by an exploitation stage. Within the variation stage he bears the costs to be able to change $C\left[\chi(r), \chi(s), \omega, T(\omega)\right] > 0$ where $\omega \in \Omega(\chi(r), \chi(s))$.He also takes advantages of some benefit to be able to change, $B^{\$}\left[\chi(r), \chi(s), \omega, T(\omega)\right] \geq 0$ (coming from curiosity, love to imagine, to search, to explore, to be able to change....).For simplification we will take this intrinsic benefit as zero. Within the exploitation stage the agent gets the extrinsic payoff $G(s, e) = tg(s, e)$.

a) if the agent chooses to he stay, he will carry out the same temporary routine $s = r = (x, t)$ as in the next period. Then, he gets the gross intrinsic benefit to stay $B^{\$}\left[\chi(r), \chi(r), \varsigma, T(\varsigma)\right] \geq 0$ for some way to stay $\varsigma \in \Omega(\chi(r), \chi(r))$ and the costs to be able to stay $C\left[\chi(r), \chi(r), \varsigma, T(\varsigma)\right] = 0$ (this costs to stay in embedded in $G(r, e) = tg(r, e)$, see before). Within the exploitation stage the agent gets the payoff $G(r, e) = tg(r, e)$.

Gross advantages to change are, over the variational and exploitation stages of the present period,

$\left[B^{\$}\left[(\chi(r), \chi(s), \omega, T(\omega)\right] - B^{\$}\left[(\chi(r), \chi(r), \varsigma, T(\varsigma)\right]\right] + t\left[g(s, e) - g(r, e)\right].$

Net advantages change: deleting opportunity costs If the agent prefers to stay, moving from $r = (x, t)$ to $r = (x, t)$ than to move from $r = (x, t)$ to $s = (y, t)$, $y \neq x$, he will not spend the time to be able to change $T(\omega) > 0$. He can use this free time to do other activities, whose payoff per unitof time is $H > 0$. Opportunity costs to move are $T(\omega)H$.Then, net advantages to change are

$A = \left[B^{\$}\left[(\chi(r), \chi(s), \omega, T(\omega)\right] - B^{\$}\left[(\chi(r), \chi(r), \varsigma, T(\varsigma)\right]\right] + t\left[g(s, e) - g(r, e)\right] - T(\omega)H.$

For simplification we will take $B^{\$}\left[(\chi(r), \chi(s), \omega, T(\omega)\right] - B^{\$}\left[(\chi(r), \chi(r), \varsigma, T(\varsigma)\right] = 0$ (to save space, we rule out intrinsic motivation).Then, net advantages to change are

$A = A(r, s, \omega, T(\omega), e) = t\left[g(s, e) - g(r, e)\right] - T(\omega)H$ for $\omega \in \Omega(\chi(r), \chi(s))$ with $s \neq r$ and

$A(r, r, \varsigma, T(\varsigma), e) = 0$, because $T(\varsigma) = 0$ when $\varsigma \in \Omega(\chi(r), \chi(r))$.

Remark: if temporary routines can repeat actions a different number of times, passing from $r = r(x) = (x, t(x)) \in \mathbf{R}$ to $s = r(y) = (y, t(y)) \in \mathbf{R}$ where $t(y)$ can be different from $t(x)$, let $r' = r(x)' = (x, t(y))$. In this case advantages to change are

$A = t(y)\left[g(s, e) - g(r', e)\right] - T(\omega)H$ for $\omega \in \Omega(\chi(r), \chi(s))$ and $s \neq r$.

The ex ante balance between motivation and resistance to change Within a period, and at the end of the consideration stage ($T(c) > 0$ units of times after the beginning of the period), the agent is able to choose (with more or less accuracy) between to stay or to move. Then, at this time, the agent must choose and give an answer to the question "should I stay, should I go". He must balance, ex ante, between advantages A and costs to be able to change C, and more generally between motivation, the utility $M = U[A]$ of advantages to change A, and the ex ante resistance to change, the desutility $R = D[C]$ of the net costs to change. The utility and desutility functions $U[.]$ and $D[.]$ are strictly increasing, with $U[0] = D[.] = 0$.Motivation and ressitance to change functions are $M(r, s, \omega, T(\omega), e) = U[A]$ where $A = A(r, s, \omega, e)$ and $C = C(\chi(r), \chi(s), \omega, T(\omega))$ where $\omega \in \Omega(\chi(r), \chi(s))$. The net motivation to change is $\Delta = M - \xi R = \Delta(r, s, \omega, T(\omega), e)$.

The coefficient $\xi = \xi(e) > 0$ is a sacrificing ratio, which puts more or less weights on resistance to change. If $0 < \xi(e) < 1$, the agent accepts some temporary sacrifices, $(1 - \xi(e))R \geq 0$. In this case the agent accepts to change even if his motivation to change does not fully compensate his resistance to change. If $\xi(e) > 1$, the agent accepts to change only if his motivation to change exceeds his resistance to change for more than $(\xi(e) - 1)R \geq 0$.

Adopting a generalized prospect formulation, where i) utility and desutility are fonctions of variations (advantages and costs to change with respect to the statu quo temporary routine r), ii) $0 \leq \pi(\omega) \leq 1$ is the probabilty that the agent can find and use the way of change $\omega \in \Omega(\chi(r), \chi(s))$ from r to s and, iii) $\psi(\pi)$ is the prospect weighting function over the probability $\pi(\omega)$, the expected net motivation (utility) to change is

$\psi(\pi(\omega))\Delta(r, s, \omega, T(\omega), e) + [1 - \psi(\pi(\omega))]\,\Delta(r, r, \varsigma, T(\varsigma), e)$
$= \psi(\pi(\omega))\Delta(r, s, \omega, T(\omega), e)$,

where $A(r, r, \varsigma, T(\varsigma), e) = 0$ and $C(\chi(r), \chi(r), \varsigma, T(\varsigma))) = 0$ for all $\varsigma \in \Omega(\chi(r), \chi(r))$ imply $\Delta(r, r, \varsigma, T(\varsigma), e) = 0$.

Then, the expected net motivation to change is $\psi\Delta = \psi(\pi(\omega))\Delta(r, s, \omega, T(\omega), e)$, where the prospect weighting function is $\psi = \psi(\pi(\omega))$. This defines ex ante worthwhile changes

$r \in \mathbf{R} \curvearrowright s \in W(r, e)$ where the "ex ante worthwhile to change set" is

$$W(r, e) = \left\{ \begin{array}{c} s \in \mathbf{R}, \text{ it exists a way to change from } r \text{ to } s, \ \omega \in \Omega(\chi(r), \chi(s)), \\ \text{such that it is worthwhile to change, } \Delta = M - \xi(e)R \geq 0 \end{array} \right\}$$

Remark 1: a change is worthwhile if resistance to change $R \leq \overline{R}$ is lower than some level $\overline{R} \geq 0$ and expected motivation to change $M \geq \overline{M}$ is higher than some level $\overline{M} \geq \xi\overline{R}$.Then, $M \geq \overline{M} \geq \xi\overline{R} \geq \xi R$.

Remark 2: a more general case can include variable preferences $U[A, e]$ and $D[C, e]$ where $e \in \mathbf{E}$ is the experience of the agent. But worthwhile to change preferences vary if we define $s \succeq_e r \Leftrightarrow s \in W(r, e)$.

"Economizing or improving enough" changes A change from a temporary routine to a new one, $r \curvearrowright s$, "improves enough" in the small if it exists $\nu > 0$ such that $A \geq \nu C$ for all $C \in \left[0, \overline{C}\right]$, $0 < \overline{C} < +\infty$ with $A = A(r, s, \omega, e)$

and $C = C(\chi(r), \chi(s), \omega, T(\omega))$.This is equivalent to say that this change is "economizing":

$0 \leq C \leq (1/\nu)A$.

"High resistance to change in the small" Consider the case where $U[.]$ is invertible. Let $\Upsilon[C] = U^{-1}[\xi D[C]]$ be the drawback of costs to be able to change function (with respect to the motivation function). Assume that i) $\Upsilon[C] \longrightarrow 0, C \longrightarrow 0$ and ii) $\Upsilon[C] \geq \nu C, \nu > 0$ for all $C \in \left[0, \overline{C}\right]$, $\overline{C} > 0$. This means that, locally, near zero, the relative resistance to change function "increases enough", at a rate higher than $\nu > 0$. In this case where the agent displays a high resistance to change in the small a worthwhile change "improves enough":

$U[A] \geq \xi D[C] \Longleftrightarrow A \geq \Upsilon[C] \geq \nu C, \nu > 0$ for all $C \in \left[0, \overline{C}\right]$ imply $A \geq \nu C$ for all $C \in \left[0, \overline{C}\right]$.

Example: the linear case where $U[A] = A$ and $D[C] = \lambda C, \lambda > 0$.

Loss aversion and high resistance to change in the small Let us show that loss aversion in the small is strongly related to high resistance to change in the small.

Definition : an agent displays loss aversion when changes relative to reference point than make things worse (losses) loom larger than improvements or gains (see Kahneman-Knetsch-Thaler, 1991. A lot of other definitions have been given, see Abdellaoui-Bleichrodt-Paraschiv 2007). In our setting $U[Z] < D[Z]$ where $Z = A = C > 0$.

A more restrictive case is loss aversion in the small: $U[Z] < D[Z]$ for $0 < Z < \overline{Z} < +\infty$.

Example 1: take $U[A] = A^{\alpha}$, $D[C] = C^{\beta}$, with $0 < \beta = 1/3 < \alpha = 1/2 \leq 1$. Then, $0 < Z^{\alpha} < Z^{\beta} \Longleftrightarrow 0 < Z^{\alpha/\beta} = Z^{3/2} < 1$ for $0 < Z < 1 = \overline{Z}$.

Then, $\Upsilon[C] = U^{-1}[\xi D[C]] = \xi^{1/\alpha} C^{\beta/\alpha} \geq \nu C$, for $0 < C < 1$ (from $\beta/\alpha > 1$), with $\nu = \xi^{1/\alpha} > 0$.This shows that, on this example, loss aversion in the small implies high resistance to change in the small.

Then, $U[A] \geq \xi D[C] \Longleftrightarrow A \geq \Upsilon[C] \Longleftrightarrow A \geq \xi^{1/\alpha} C^{\beta/\alpha} = R^r[C] \geq \nu C$, for $0 < C < 1$ (from $\beta/\alpha > 1$).

In this case a worthwhile change "improves enough" when the agent displays loss aversion and costs to be able to change are small enough.

Example 2: $U[A] = A$ and $D[C] = \lambda C, \lambda > 1$.

9 The consideration of worthwhile changes: convergence

In this section we suppose that, each step, the agent, starting from a temporary routine $r = (x, t) \in \mathbf{R}_t$, and his related experience $e \in \mathbf{E}$, is able to find an acceptable change $s \in \Phi(r, e)$ where $s = (y, t) \in \mathbf{R}_t$ (see Soubeyran A, 2010b, for the use of acceptable sets of change). In the present paper we take as a more specific acceptable set of change the worthwhile to change set $\Phi(r,e) = W(r,e)$.

Suppose high resistance to change in the small.Then, the worthwhile to change set at (r,e) is

$$W(r,e) = \left\{ \begin{array}{c} s \in \mathbf{R}_t, \text{ it exists } \omega \in \Omega(\chi(r),\chi(s)), \text{ such that} \\ A - \Upsilon \geq 0 \end{array} \right\}$$

where $A = t\left[g(s,e) - g(r,e)\right] - T(\omega)H$, and $\Upsilon = U^{-1}\left[\xi D\left[C\right]\right]$ with $C = C(\chi(r),\chi(s),\omega,T(\omega))$.

Consideration sets and exploration sets Each step, starting from a temporary routine $r = (x,t) \in \mathbf{R}_t$, and his related experience $e \in \mathbf{E}$, we suppose that among the subset $F(r,e) \subset \mathbf{R}_t$ of temporary routines $s \in F(r,e)$ that the agent considers, a subset $\Gamma(r,e) \subset F(r,e)$ of them defines a worthwhile change $s \in W(r,e)$.This means that, each step, the intersection $\Gamma(r,e) = F(r,e) \cap W(r,e)$ of the consideration set $F(r,e)$ and the worthwhile to change set $W(r,e)$ is not empty, where $r \in \Gamma(r,e)$ for all $r \in \mathbf{R_t}$ and all $e \in \mathbf{E}$.The definition of the consideration set has been given in Soubeyran A (2010.b). It represents the subset of temporary routines $s = (y,t)$ that the agent considers more or less seriously in a first stage. In a second stage, the exploration stage, the agent examines more seriously each of these temporary routine, rejects some of them and keeps for even more serious examination a last subset. In our case this last subset is the exploration set $\Gamma(r,e) \subset F(r,e)$. More serious examination means to consider more criteria (a more extensive examination), or to consider more deeply a given criteria (a more intensive examination) to imagine new temporary routines s, to imagine and estimate the capabilities $\chi(s) \in \boldsymbol{\chi}$ to do them and their related per action payoff $g(s,e) \in \mathbb{R}$.

Remark: our worthwhile to change process does not suppose local optimization within the consideration set, although this can be the case.

Learning by doing and loving by repetition: the role of experience Consider a considered worthwhile to change process $r_{n+1} \in \Gamma(r_n,e_n)$, $n \in N$ where $r_n \in \Gamma(r_n,e_n)$ for all $n \in N$. Let $e_{n+1} = \Psi(r_{n+1},e_n) \in \mathbf{E}$ be the experience coming from having done the temporary routine r_{n+1}, with the given experience $e_n \in \mathbf{E}$.This means that $e_1 = \Psi(r_1,e_0), e_2 = \Psi(r_2,e_1) = \Psi(r_2,\Psi(r_1,e_0))$, ...etc.....More generally, $e_s = \Psi(s,e_r)$ is the experience of the agent after having done the temporary routine s, given his experience e_r after having done the temporary routine r.

Hypothesis: there is learning by doing or loving by repetition: $e_s = \Psi(s,e_r) \Longrightarrow g(s,e_s) \geq g(s,e_r)$. This means that doing a temporary routine s for the second time improves the per action payoff.

Convergence of worthwhile changes We give conditions under which a worthwhile to change process converges.

Theorem 1: consider a given capability map $\chi(.) : r \in \mathbf{R}_t \longrightarrow \chi(r) \in \boldsymbol{\chi}_r$. Assume that, i) the capability map is injective and $\chi(s) \neq \chi(r) \Longrightarrow$

$\underline{C}(\chi(r), \chi(s)) > 0$, ii) the quasi metric space $\mathbf{R}_t$ is left complete, iii) per action payoffs are bounded above, $g(r,e) \leq \overline{g} < +\infty$ for all $r \in \mathbf{R}_t$,$e \in \mathbf{E}$, iv) there is learning by doing or loving by repetition, v) there is high resistance to change in the small, vi) rate of compensation are high enough: $\xi(r) \geq \underline{\xi} > 0$ for all $r \in \mathbf{R}_t$, vii) assume that the agent considers a worthwhile to change process $r_{n+1} \in \Gamma(r_n, e_n)$, $n \in N$ where $r \in \Gamma(r,e)$ for all $r \in \mathbf{R}_t$ and $e \in \mathbf{E}$.Then,

a) avantages to change $t\left[g(r_{n+1}, e_n) - g(r_n, e_n)\right] - T(\omega_{n+1})H$ go to zero, the per action payoff converges: $g_n \longrightarrow g^*, n \longrightarrow +\infty$, resistance to change goes to zero, the sum of advantages to change is finite, as well as the sum of resistance to change.

b) the length of the variational stage goes to zero, $T(\omega_n) \longrightarrow 0$,$n \longrightarrow +\infty$, and the total time spend to change is finite, $\Sigma_{n \in \mathbf{N}} T(\omega_n) < +\infty$.

c) worthwhile to change temporary routines $r_{n+1} \in \Gamma(r_n, e_n), n \in N$ converge in term of the resistance to change quasi metric, $r_n \longrightarrow r^* \in \mathbf{R}$,$n \longrightarrow +\infty$.

d) the agent makes small steps if the distance $d(r,s) \geq 0$ between two temporary routines satisfies $l(r,s) \geq \kappa d(r,s), \kappa > 0$, for all $r, s \in \mathbf{R}_t$.

and the length of the worthwhile to change path is finite.

e) convergence is in finite time if there are fixed costs to change, $\underline{C}(\chi(r), \chi(s)) \geq \underline{C} > 0$ for all $s \neq r, r, s \in \mathbf{R}_t$.

Proof:

Given the capability map $\chi(.) : r \in \mathbf{R}_t \longrightarrow \chi(r) \in \boldsymbol{\chi}_r$, let us define $l(r,s) = \underline{C}(\chi(r), \chi(s)) \geq 0$. Hypothesis i) shows that $l(r,s) = \underline{C}(\chi(r), \chi(s)) = 0 \Longrightarrow s = r$ and $l(r,r) = \underline{C}(\chi(r), \chi(r)) = 0$.Then, the space $\mathbf{R}_t$ of temporary routines $r \in \mathbf{R}_t$ is a quasi metric space endowed with the quasi distance $l(r,s)$. The triangular inequality has been shown before (see the section on resistance to change).

Assume that the agent considers a worthwhile to change process $r_{n+1} \in \Gamma(r_n, e_n)$, $n \in N$ where $r_n \in \Gamma(r_n, e_n)$ for all $n \in N$.

Let $r_n = r(x_n) = (x_n, t), \xi(r_n) = \xi_n > 0, \omega_{n+1} \in \Omega(\chi(r_n), \chi(r_{n+1})), g(r_n, e_n) = g_n$.

$A_{n,n+1} = t\left[g(r_{n+1}, e_n) - g(r_n, e_n)\right] - T(\omega_{n+1})H$

and $C_{n,n+1} = C(\chi(r_n), \chi(r_{n+1}), \omega_{n+1}, T(\omega_{n+1}))$

Then, $r_{n+1} \in \Gamma(r_n, e_n) \Longleftrightarrow A_{n,n+1} \geq \Upsilon_{n,n+1} \geq 0$, where

$\Upsilon_{n,n+1} = U^{-1}\left[\xi_n D\left[C_{n,n+1}\right]\right]$.This implies

$A_{n,n+1} \geq \Upsilon_{n,n+1} \geq 0 \Longrightarrow A_{n,n+1} = t\left[g(r_{n+1}, e_n) - g(r_n, e_n)\right] - T(\omega_{n+1})H \geq 0 \Longrightarrow t\left[g(r_{n+1}, e_n) - g(r_n, e_n)\right] \geq T(\omega_{n+1})H \geq 0$

Then (hypothesis iv), $g(r_{n+1}, e_{n+1}) \geq g(r_{n+1}, e_n)$ implies $t\left[g_{n+1} - g_n\right] \geq T(\omega_{n+1})H \geq 0$. This gives $g_{n+1} \geq g_n$, for all $n \in \mathbf{N}$.Furthermore $g_n \leq \overline{g} < +\infty$ for all $n \in \mathbf{N}$ (hypothesis iii). Then, the improving and bounded above sequence $\{g_n\}$ converges: $g_n \longrightarrow g^*, n \longrightarrow +\infty$. This implies that $g_{n+1} - g_n \longrightarrow 0, n \longrightarrow +\infty$. Then, the lenght of the variational stage vanishes: $T(\omega_{n+1}) \longrightarrow 0, n \longrightarrow +\infty$.Furtermore the inequalities $0 \leq \Sigma_{n \in \mathbf{N}} T(\omega_n)H \leq t\Sigma_{n \in \mathbf{N}}\left[g_{n+1} - g_n\right] \leq t\left[\overline{g} - g_0\right] < +\infty$ imply $\Sigma_{n \in \mathbf{N}} T(\omega_n) \leq t\left[\overline{g} - g_0\right] < +\infty$.

The inequality $t\left[g_{n+1} - g_n\right] \geq A_{n,n+1} \geq 0$ implies that $A_{n,n+1} \longrightarrow 0, n \longrightarrow +\infty$. The inequality $A_{n,n+1} \geq \Upsilon_{n,n+1} \geq 0$ shows that $\Upsilon_{n,n+1} \longrightarrow 0, n \longrightarrow +\infty$.

Then, for all $\varepsilon > 0$, it exists $m(\varepsilon) \in \mathbf{N}$ such that for all $n \geq m(\varepsilon), 0 \leq \Upsilon_{n,n+1} = U^{-1}\left[\xi_n D\left[C_{n,n+1}\right]\right] \leq \varepsilon$.This implies that, for all $n \geq m(\varepsilon)$ (using hypothesis vi)) we have $\underline{\xi} D\left[C_{n,n+1}\right] \leq \xi_n D\left[C_{n,n+1}\right] \leq U\left[\varepsilon\right]$,which implies itself $0 \leq C_{n,n+1} \leq \overline{C}$ $n \geq m(\varepsilon)$,where $\overline{C} = D^{-1}\left[(1/\underline{\xi})U\left[\varepsilon\right]\right] > 0$.

Using hypothesis v) and $0 \leq C_{n,n+1} \leq \overline{C}$ imply the inequality

$t\left[g_{n+1} - g_n\right] \geq A_{n,n+1} \geq \Upsilon_{n,n+1} \geq \nu C_{n,n+1}, \nu > 0$ which implies $t\left[g_{n+1} - g_n\right] \geq \nu C_{n,n+1} \geq 0$ (*) for all $n \geq m(\varepsilon)$.Then, i) costs to change move to zero, $C_{n,n+1} \longrightarrow 0, n \longrightarrow +\infty$, ii) the quasi distance between two consecutive temporary routines vanishs, because

$C_{n,n+1} = C(\chi(r_n), \chi(r_{n+1}), \omega_{n+1}, T(\omega_{n+1})) \geq \underline{C}(\chi(r_n), \chi(r_{n+1})) = l(r_n, r_{n+1}) \geq 0$ for all $n \in \mathbf{N}$.

Adding the inequalities $t\left[g_{n+1} - g_n\right] \geq \nu C_{n,n+1} \geq \nu l(r_n, r_{n+1})$, for all $n \geq m(\varepsilon) \in \mathbf{N}$ gives

$+\infty > t\left[\overline{g} - g_0\right] \geq t\left[g_{h+1} - g_0\right] \geq \nu \Sigma_{n=m(\varepsilon)}^{h} l(r_n, r_{n+1}) \geq 0$ for all $h \in \mathbf{N}$.

This shows that the sequence of temporary routines $r_n, n \in N$, converges in the left sequentially complete quasi metric space $\mathbf{R}_t$.

Finally, let $d(r,s)$ be a distance between two temporary routines. If $l(r,s) \geq \kappa d(r,s), \kappa > 0$ for all $r, s \in \mathbf{R}_t$, the inequality

$l(r_n, r_{n+1}) \geq \kappa d(r_n, r_{n+1}), \kappa > 0, n \in \mathbf{N}$, shows that the distance between two consecutive temporary routines vanishs as well: $d(r_n, r_{n+1}) \longrightarrow 0, n \longrightarrow +\infty$.The agent makes small steps. Convergence in term of distance d follows from the inequalities

$+\infty > t\left[\overline{g} - g_0\right] \geq t\left[g_{h+1} - g_0\right] \geq \nu \Sigma_{n=m(\varepsilon)}^{h} l(r_n, r_{n+1}) \geq \nu \Sigma_{n=n(\varepsilon)}^{h} l(r_n, r_{n+1})$ for all $h \in \mathbf{N}$.

Convergence is in finite time if there are fixed costs to change. This comes from summing the previous inequalities (*) given in the proof,

$t\left[g_{n+1} - g_n\right] \geq \nu C_{n,n+1}, \nu > 0$ for $n \geq m(\varepsilon)$.Then,

$+\infty > t\left[\overline{g} - g_0\right] \geq t\left[g_{h+1} - g_{m(\varepsilon)}\right] \geq \nu \Sigma_{n=m(\varepsilon)}^{h+1} C_{n,n+1} \geq \nu \underline{C}\left[(h+1) - m(\varepsilon)\right]$ which goes to infinity if $h \longrightarrow +\infty$. The result follows.

Remark 1: This theorem shows that the time spend to pass from a temporary routine to a new one, $\Lambda(\omega_n)$, vanishs.Then, worthwhile to change temporary routines become more and more the same, and the time spend to be able to change disappears (learning vanishs). The succession of repeated actions is routinized. When $l(r,s) \geq \kappa d(r,s), \kappa > 0$ for all $r, s \in \mathbf{R}_t$,costs to be able to change are high enough. There is a lot of inertia.

Remark 2 : by definition, a sequence $\{r_n\}$ in $\mathbf{R}$ is said to be a left $\varepsilon > 0$ Cauchy sequence if for each $\varepsilon > 0$, there exists an $m(\varepsilon) \in N$ such that $\underline{R}(r_p, r_q) < \varepsilon$ for all $q \geq p \geq m(\varepsilon)$. A quasi metric space is left ε sequentially complete if each left ε Cauchy sequence is convergent (for definitions, see Ume, 2002, and for a survey on quasi metric spaces, see Mennucci, 2007).

10 Aspiration and consideration traps (permanent routines).

Traps The limit temporary routine of the previous section is not necessarily a permanent routine to be defined now (an aspiration and, or, a worthwhile to change trap). Let $r_{n+1} \in \Gamma(r_n, e_n), n \in N$ be a considered succession of worthwhile to change temporary routines. A main question: is what can be the end of such a process ? The experience of the agent at $r^* = (x^*, t)$ being $e^* = e(r^*)$, it can be, if this is the case,

i) an aspiration trap, ie a permanent routine $r^* \in \mathbf{R}_t$, where the motivation to change disappears because aspiration gap is zero, $\widehat{a}_\Gamma(r^*, e^*) = \widehat{g}_\Gamma(r^*, e^*) - g(r^*, e^*) = 0$, or (and)

ii) a worthwhile to change trap $W(r^*, e^*) = \{r^*\}$ where the agent stops to consider other temporary routine different from r^* because all of them are not worthwile. In some case the two coincide.

The two notions are different. Being at a worthwhile to change trap, the agent does not consider temporary routines different from the present one. He stops to imagine a new situation, to explore. Worthwhile to change traps are aspiration traps which are also ability traps where the agent stops learning (does not acquire new knowledge and new capabilities). For an aspiration trap the agent has no more the motivation to change. Habits and routines share the two properties. They are both aspiration and worthwhile to change traps. Our variational formulation explains why it has been so difficult to give a precise modelisation of these concepts, the rich litterature on these topics being quite confusing, with no clear definitions and properties of habits and routines (Lazaric, 2000).

Variational lower semicontinuity A numerical funtion $\varphi(.) :: z \in \mathbf{Z} \longrightarrow \varphi(.) \in \mathbb{R}$ is variationally lower semicontinuous (lsc) at $z^* \in \mathbf{Z}$ if it exists $\eta > 0$ such that

if $\lim_{n \longrightarrow +\infty} \varphi(z_n) = \varphi^*$ when $z_n \longrightarrow z^*$, then, $\varphi^* \geq \eta\varphi(z^*)$.To understand this definition, consider the aspiration gap function $\varphi(z) = \widehat{a}(z) \in \mathbb{R}_+$.where $z = (r, e) \in \mathbf{Z}$. Then, the aspiration gap function is variationally lsc at z^* if it exists $\eta > 0$ such that if $\lim_{n \longrightarrow +\infty} \widehat{a}(z_n) = a^*$ when $z_n \longrightarrow z^*$, then, $a^* \geq \eta\widehat{a}(z^*)$.This means that if aspiration gap $\widehat{a}(z_n)$ converges to some value a^*, it does not explose too much in the limit (if η is small).This new concept of lower semi continuity generalizes the "inf sequentially lsc at z^*" (Aruffo-Bottaro, 2010) which itself generalizes "sequentially lsc from above at z^*" (Chen-Cho-Yang, 2002) which itself generalizes the traditional concept of lsc.

Remark: Let (Z, d) be a metric space. A function $\varphi(.) :: z \in \mathbf{Z} \longrightarrow \varphi(.) \in \overline{\mathbb{R}}$ is "lsc at z^* " if, for any sequence $\{z_n\}$ in $\mathbf{Z}$ with $z_n \longrightarrow z^*$, $\varphi(z^*) \leq \underline{\lim}_{n \longrightarrow +\infty} \inf \varphi(z_n)$. Following (Chen-Cho-Yang, 2002) such a function $\varphi(.)$ is "lsc from above at $z^* \in \mathbf{Z}$" if, for any sequence $\{z_n\}$ in $\mathbf{Z}$ with $z_n \longrightarrow z^*$, and $\varphi(z_0) \geq \varphi(z_1) \geq ... \geq \varphi(z_n) \geq ....$, then, $\lim_{n \longrightarrow +\infty} \varphi(z_n) = \varphi^*$ exists and $\varphi(z^*) \leq \varphi^* = \lim_{n \longrightarrow +\infty} \varphi(z_n)$. For Aruffo-Bottaro (2010) the function $\varphi(.)$ is

" inf-sequentially lsc at z^*" if, for any sequence $\{z_n\}$ in $\mathbf{Z}$ with $z_n \longrightarrow z^*$ and $\lim_{n\longrightarrow+\infty} \varphi(z_n) = \inf \varphi(.)$, then, $\varphi(z^*) = \inf \varphi(.)$.

11 The goal pursuit model

In this section we suppose that, each step $n \in N$, the agent tries to satisfice within the considered portion of the "worthwhile to change" set $\Gamma(r_n, e_n)$. Let us be more explicit. The period is divided in three stages: consideration, variation and repetition (exploitation). In the litterature the two first represent "exploration". For us exploration is only one sub-stage of the consideration stage.

The consideration stage: finding satisficing worthwhile changes 1) the inventory stage: given the initial temporary routine $r_n \in R$ which has been done the previous period and his the related experience $e_n \in E$, the agent considers the per action payoff $g(r_n, e_n)$ to do again this temporary routine this period.

2) the consideration stage: the agent considers more or less seriously a "consideration set" $F(r_n, e_n) \subset \mathbf{R}_t$.

3) the exploration stage: the agent explores (examines more deeply) the "worthwhile to change" portion $\Gamma(r_n, e_n) = F(r_n, e_n) \cap W(r_n, e_n)$.

4) the aspiration stage: the agent sets an aspiration level $\widehat{g}_\Gamma(r_n, e_n) \geq g(r_n, e_n)$, or sets an aspiration gap $\widehat{a}_\Gamma(r_n, e_n) = \widehat{g}_\Gamma(r_n, e_n) - g(r_n, e_n) \geq 0$,

5) the continuation stage: i) If $\widehat{a}_\Gamma(r_n, e_n) = 0$, the agent stops there and do again the temporary routine r_n, ii) If $\widehat{a}_\Gamma(r_n, e_n) > 0$ the agent, as we suppose, wants to fill a portion of his aspiration gap. He considers a variation, a change from the starting temporary routine to a new temporary routine, $r_n \curvearrowright r_{n+1}$, to improve his per action payoff from $g(r_n, e_n)$ to $g(r_{n+1}, e_n) > g(r_n, e_n)$. His per action advantage to change will be $a(r_n, r_{n+1}, e_n) = g(r_{n+1}, e_n) - g(r_n, e_n) > 0$.

6) the goal setting stage: if $\widehat{a}_\Gamma(r_n, e_n) > 0$, the agent sets a satisficing level $\widetilde{g}(r_n, e_n) \in \,]g(r_n, e_n), \widehat{g}_\Gamma(r_n, e_n)[$, strictly higher than the present per action payoff, and strictly lower than the aspiration level. This is equivalent to set a strictly positive satisficing gap

$\widetilde{a}(r_n, e_n) = \widetilde{g}(r_n, e_n) - g(r_n, e_n) > 0$, with $0 < \widetilde{a}(r_n, e_n) < \widehat{a}_\Gamma(r_n, e_n)$.

7) the satisficing stage: the agent wants to satisfices (to "improve enough"). Then, he will have to explore within the worthwhile to change portion $\Gamma(r_n, e_n)$ of the consideration set to be able to find a new temporary routine $r_{n+1} \in \Gamma(r_n, e_n)$ such that $g(r_n, e_n) < \widetilde{g}(r_n, e_n) \leq g(r_{n+1}, e_n) \leq \widehat{g}_\Gamma(r_n, e_n)$. This is equivalent to set a satisficing ratio $0 < \beta(r_n, e_n) < 1$ and to try to find a new temporary routine $r_{n+1} \in \Gamma(r_n, e_n)$ which satisfices, ie such that $a(r_n, r_{n+1}, e_n) \geq \beta(r_n, e_n)\widehat{a}(r_n, e_n)$. In this case the satisficing level will drive the exploration process.

The variational and repetitive stages Within the variation stage the agent must build the capabilities $\chi(r_n)$ to be able to do the satisficing temporary routine r_{n+1}.Within the exploitation (repetitive) stage the agent performs the temporary routine r_{n+1}.

Degree of fit and feasible satisficing changes. The degree of fit $\alpha(r_n, e_n) > 0$ between the aspiration gap and the consideration set of actions is defined by the equality $\widehat{a}_\Gamma(r_n, e_n) = \alpha(r_n, e_n)\overline{a}_\Gamma(r_n, e_n)$

where $\overline{a}_\Gamma(r_n, e_n) = \sup\{a(r_n, s, e_n), s \in \Gamma(r_n, e_n)\}$ is the true aspiration gap (Soubeyran A, 2010.b).The satisficing condition $a(r_n, r_{n+1}, e_n) \geq \beta(r_n, e_n)\widehat{a}_\Gamma(r_n, e_n)$, $0 < \beta(r_n, e_n) < 1$ and the fit condition give the feasible satisficing condition $a(r_n, r_{n+1}, e_n) \geq \theta_\Gamma(r_n, e_n)\overline{a}_\Gamma(r_n, e_n)$

which is feasible if $0 < \theta_\Gamma(r_n, e_n) = \alpha(r_n, e_n)\beta(r_n, e_n) < 1$.

The agent "satisfices enough" if it exists $0 < \underline{\theta} < 1$ such that $0 < \underline{\theta} \leq \theta_\Gamma(r_n, e_n) = \alpha(r_n, e_n)\beta(r_n, e_n) < 1$.

Remark: at the mathematical level the variational problem is the find an approximate solution of each state depend optimization problem ε_n

$- \sup\{\ a(r_n, s, e_n), s \in \Gamma(r_n, e_n)\}, n \in N$

where $a(r_n, s, e_n) = g(s, e_n) - g(r_n, e_n)$.This is a course pursuit problem. At stage n, an "epsilon- sup solution" of the problem is any $\widetilde{s} \in \Gamma(r_n, e_n)$ such that

$a(r_n, \widetilde{s}, e_n) \geq \sup\{\ a(r_n, s, e_n), s \in \Gamma(r_n, e_n)\} - \varepsilon_n \geq 0, \varepsilon_n > 0$.

Then, the variation $r_n \curvearrowright \widetilde{s}$ defines a satisficing change. If $\varepsilon_n = 0$, it represents an optimal change.

12 Goal pursuit, using worthwhile transitions between temporary routines.

The main variational problems As seen before let us summarize the goal pursuit model. Each step, the agent, starting from having done a temporary routine, either does not consider other ones, or considers new ones. In the later case he sets an aspiration gap related to a consideration set. At this stage he imagines and examines new repeated actions, and the capabilities to do them. Then, he sets a satisficing level, and tries to find a repeated action which can reach this level. Having done that, he must build the capabilites to do this repeated action. Finally he performs the repeated action. And so on.....His utility function changes with the repeated action along a course pursuit between means and ends.

Theorem 1 has given conditions under which,

a) avantages to change go to zero, the per action payoff converges, resistance to change goes to zero, the sum of advantages to change is finite, as well as the sum of resistance to change.

b) the length of the variational stage goes to zero, and the total time spend to change is finite.

c) worthwhile to change temporary routines converge in term of the resistance to change quasi metric.

d) the agent makes small steps and the length of the worthwhile to change path is finite, if the distance between two temporary routines is lower than a fraction of the distance between the capabilities to do them.

e) the total time spend to move is finite if there are fixed costs to change.

Consider now a goal pursuit described in section 9 (see Soubeyran A, 2010b for more details on this process). If the agent uses as acceptable change worthwhile changes, let us show that,

f) if the agent "satisfices enough", aspiration gaps (expected and true) vanish.

g) the agent, using worthwhile changes, uses "economizing enough" changes. This implies that temporary routines converge to some point (theorem 1). This point is an aspiration trap if there is some variational lower semicontinuity assumption.

h) Finally we will examine when an aspiration trap is a worthwhile to change trap.

Hypothesis. Let us examine an agent which considers a worthwhile to change process $r_{n+1} \in \Gamma(r_n, e_n)$, $n \in N$ where,

1) hypothesis of theorem 1 are valid.

2) the agent is able to "satisfice enough" each step: it exists $0 < \underline{\theta} < 1$ such that

$0 < \underline{\theta} \leq \theta_\Gamma(r_n, e_n) = \alpha(r_n, e_n)\beta(r_n, e_n) < 1$.

3) variational lower semicontinuity (see section 8)

4) $\overline{a}_\Gamma(r, e) \geq \tau \overline{a}_W(r, e)$, with $0 < \tau \leq 1$, where, as said before, $\Gamma(r, e) = F(r, e) \cap W(r, e) \neq \phi$ for all $(r, e) \in \mathbf{R}_t \times \mathbf{E}$ is the non empty considered portion of the worthwhile to change set, $\overline{a}_W(r, e) = \sup\{a(r, s, e), s \in W(r, e)\}$ and $\overline{a}_\Gamma(r, e) = \sup\{a(r, s, e), s \in \Gamma(r, e)\}$ are the true aspiration gap of the gross per action advantage to change function $a(r, s, e) = g(s, e) - g(r, e)$ over these two sets. This hypothesis means that the agent considers and explores a large enough portion of the worthwile to change set.

The second variational theorem: the disparition of aspiration gaps. f) **extinction of the aspiration gap**. Assume 1) and 2). Then, the agent can choose, each step, a worthwhile and "satisficing enough" change, and the aspiration gaps (both estimated and true) vanish along such a process, $\widehat{a}_\Gamma(r_n, e_n) \longrightarrow 0, n \longrightarrow +\infty$ and $\overline{a}_\Gamma(r_n, e_n) \longrightarrow 0, n \longrightarrow +\infty$.

g) **convergence to an aspiration trap, making small steps.** Assume 1) 2) 3). Then the conclusion of theorem 1 follows. Furthermore a path of worthwhile to change and "satisficing enough" temporary routines converges to an aspiration trap (existence).

h) **when an aspiration trap is a worthwhile to change trap**: assume 1), 2), 3), 4). Then, any aspiration trap is also a worthwhile to change trap: $\overline{a}_\Gamma(r^*, e^*) = 0 \Longrightarrow W(r^*, e^*) = \{r^*\}$.

Proof: consider a considered worthwhile to change process $r_{n+1} \in \Gamma(r_n, e_n)$, $n \in N$. Using the notation of theorem 1,

$r_{n+1} \in \Gamma(r_n, e_n) \iff \Delta_{n,n+1} = A_{n,n+1} - \xi_n C_{n,n+1} \geq 0 \iff A_{n,n+1} = \Delta_{n,n+1} + \xi_n C_{n,n+1}$ with $\Delta_{n,n+1} \geq 0$, where

advantages to change are $A_{n,n+1} = t\left[g(r_{n+1}, e_n) - g(r_n, e_n)\right] - T(\omega_{n+1})H$,

costs to be able to change are $C_{n,n+1} = C(\chi(r_n), \chi(r_{n+1}), \omega_{n+1}, T(\omega_{n+1}))$, the way of change is $\omega_{n+1} \in \Omega(\chi(r_n), \chi(r_{n+1}))$

with $g(r_n, e_n) = g_n$ and $\xi(r_n) = \xi_n > 0$.

Consider point f). Assume (hypothesis 2) that the agent is able to "satisfice enough". Suppose that he chooses a "satisficing enough" worthwhile change $r_{n+1} \in \Gamma(r_n, e_n)$ such that $a(r_n, r_{n+1}, e_n) \geq \theta_\Gamma(r_n, e_n)\overline{a}_\Gamma(r_n, e_n) \geq 0$.This change is feasible if

$0 < \theta_\Gamma(r_n, e_n) = \alpha(r_n, e_n)\beta(r_n, e_n) < 1$ where $a(r_n, r_{n+1}, e_n) = g(r_{n+1}, e_n) - g(r_n, e_n)$. This change is worthwile. Then,

$t\left[g_{n+1} - g_n\right] \geq ta(r_n, r_{n+1}, e_n) \geq A_{n,n+1} \geq 0$.This implies

$t\left[g_{n+1} - g_n\right] \geq t\theta_\Gamma(r_n, e_n)\overline{a}_\Gamma(r_n, e_n) \geq t\underline{\theta}\overline{a}_\Gamma(r_n, e_n) \geq 0$.

Then, $g_{n+1} - g_n \longrightarrow 0, n \longrightarrow +\infty$, implies that the true aspiration gaps goes to zero: $\overline{a}_\Gamma(r_n, e_n) \longrightarrow 0, n \longrightarrow +\infty$. Then, the expected aspiration gap goes to zero, $\widehat{a}_\Gamma(r_n, e_n) \longrightarrow 0, n \longrightarrow +\infty$.

Consider point g). Assume hypothesis 1. Then, the conclusin of theorem 1 are valid. Worthwhile to change temporary routines converge to a point. This point is an aspiration trap because of the variational lower semicontinuity hypothesis 3.

Finally, examine point h). Hypothesis 4) gives $\overline{a}_\Gamma(r_n, e_n) \geq \tau\overline{a}_W(r_n, e_n) \geq 0$ for all $n \in \mathbf{N}$ (the proof works if it exists $m \in \mathbf{N}$ such that this is true). Then, $\overline{a}_\Gamma(r_n, e_n) \longrightarrow 0, n \longrightarrow +\infty$ implies $\overline{a}_W(r_n, e_n) \longrightarrow 0, n \longrightarrow +\infty$.On an other let us show that $\overline{a}_W(r, e) \geq (\xi/t)radiusW(r, e) \geq 0$, where $radiusW(r, e) = \sup\left\{l(r, s), s \in W(r, e)\right\}$ and $l(r, s) = \underline{C}(\chi(r), \chi(s)) \geq 0$ is a quasi distance between the temporary routines r and s and $a(r, s, e) = g(s, e) - g(r, e)$. The proof works as follows: $s \in W(r, e)$ implies $a(r, s, e) \geq (\xi/t)l(r, s) \geq 0$, which comes from the inequalities,

$ta(r, s, e) \geq A(r, s, e, T(\omega)) = t\left[g(s, e) - g(r, e)\right] - T(\omega)H \geq$

$\xi C(\chi(r), \chi(s), \omega, T(\omega)) \geq \xi l(r, s) \geq 0$.

Then, take the supremum for both sides of the inequality over all $s \in W(r, e)$.Then, $\overline{a}_W(r_n, e_n) \geq (\xi/t)radiusW(r_n, e_n) \geq 0$ and $\overline{a}_W(r_n, e_n) \longrightarrow 0, n \longrightarrow +\infty$ imply that $radiusW(r_n, e_n) \longrightarrow 0, n \longrightarrow +\infty$.In the limit $0 = \overline{a}_\Gamma(r^*, e^*) \geq \tau\overline{a}_W(r^*, e^*) \geq \tau(\xi/t)radiusW(r^*, e^*) \geq 0$ shows that $radiusW(r^*, e^*) = 0$. Then, $W(r^*, e^*) = \left\{r^*\right\}$.

The worthwhile to change set shrinks to the permanent routine r^*.In this case where the agent "considers and explores enough" an aspiration trap $\overline{a}_\Gamma(r^*, e^*) = 0$ is a worthwhile to change trap $W(r^*, e^*) = \{r^*\}$.

13 No regret, consideration costs and consideration traps

Let $sizeF \geq 0$ be the size of the consideration set $F = F(r,e) \subset \mathbf{R}_t$ where $sizeF = 0$ if $cardF \leq 1$.Let $L\left[sizeF(r,e)\right] \geq 0$ be the consideration costs which increases with its size and such that $L\left[sizeF\right] \longrightarrow 0$ if $sizeF \longrightarrow 0$. Each period, these costs are spend in the first stage of the period. At the end of this consideration stage, they allow the agent to be able to choose between "stay" and repeat the previous temporary routine r or "move" to a new temporary routine s, balancing between known motivation and resistance to change. If the agent finds that it is preferable to stay, these consideration costs seem to have been spend for nothing (they are sunk). Then, to try to be bounded rational is costly.

If the agent chooses to change, he will feel no regret to have spend costs to consider change when his net motivation to change $\Delta = M - \xi R$ is higher than his consideration cost $L\left[sizeF(r,e)\right]$, ie $\Delta = M - \xi R \geq L\left[sizeF(r,e)\right] \geq 0$. Along a convergent worthwhile process of change

$r_{n+1} \in \Gamma(r_n, e_n), n \in N$ where $(r_n, e_n) \longrightarrow (r^*, e^*), n \longrightarrow +\infty$, and $\Delta_n \longrightarrow 0, n \longrightarrow +\infty$, the no regret inequality $\Delta_n \geq L\left[sizeF(r_n, e_n)\right] \geq 0, n \geq m$ implies that consideration costs must go to zero, $L\left[sizeF(r_n, e_n)\right] \longrightarrow 0, n \longrightarrow +\infty$. This implies that the size of the consideration set goes to zero, $sizeF(r_n, e_n) \longrightarrow 0, n \longrightarrow +\infty$. This shows that the agent considers less and less new temporary routines as the worthwhile process of change evolves. This is a third other aspect of a routinization process. If, $\lim_{n \longrightarrow +\infty} sizeF(r_n, e_n) \geq sizeF(r^*, e^*) \geq 0$ (a variational lower semicontinuity condition), the agent moves to a consideration trap r^* where $F(r^*, e^*) = \{r^*\}$ and his experience is e^*.

If consideration costs represent, each period, some minimum fixed costs $L\left[sizeF(r_n, e_n)\right] \geq \underline{L} > 0$, a worthwhile to change process will stop in a finite number of periods: $\Delta_n \longrightarrow 0, n \longrightarrow +\infty$ and the no regret inequalities $\Delta_n \geq L\left[sizeF(r_n, e_n)\right] \geq \underline{L} > 0, n \in \mathbf{N}$ show this result.

Remark: consideration costs can generate regret. Several questions remain: how contentment and regret coming from past successes and failures impact on the formation of aspiration levels and the evolution of his motivation ? If the agent is close to the past aspiration level he will set a higher aspiration level or will feel satiation. This depends of his personality traits which determine how he forms aspiration gaps i$\widehat{a}(y) = F\left[\widehat{a}(x), \theta_\Gamma(x), \text{ past succes and failures....}\right]$.....

14 Microfoundations for a general theory of change

Theories of change. They include behavioral economics (the old theory of bounded rationality, procedural rationality and satisficing, Simon,1955,1956), the new prospect theory (Kahneman-Tversky, 1979), capability based theory (resource -competence-knowledge-cognitive-learning based), evolutionary economics and the economics of transition (Schumpeter 1950, Nelson-Winter 1982

), the psychology of change (motivation and resistance to change theories), and local search optimization algorithms.

Variational rationality. **A variational model of change** defines a course pursuit between means and ends, moving from old to news. It includes

i) a state space of means (actions, capabilities to do them, experience),

ii) a state space of ends (utilities on variations, feelings, aspirations, goals,...., emotions),

iii) a model of change, including a list of activities (operations) and several stages where these activities are performed,

a) changing activities: 1) aspiration and consideration stage: aspiration feelings, consideration, (goal setting, imagination, exploration, evaluation), 2) decision to change, balancing motivation and resistance to change, stability and change, 3) variational stage: problem solving, learning by thinking, variations, implementation of change, capability building, dynamic capabilities, creation, invention, innovation.

b) repetitive activities: exploitation stage (temporary habits and routines, learning by doing, the competency trap),

The variational man is a creature of temporary habits and routines, entwinning repetitive and changing activities, moving from temporary habits and routines to new ones, who,

1) transforms, each step, his frustration feeling, coming from the existence of a temporary aspiration gap, into the motivation to fill a portion of it,

2) considers changes (imagine, explore, evaluate, and sets intermediate goals to try to satisfice without too much sacrificing.

3) chooses between change or stay, balancing, each step, between motivation and resistance to change (using worthwhile changes).

4) implements changes, building new capabilities, deleting old, learning, to be able to temporary satisfice.

vi) chooses the timing of change (punctuated, regular-irregular, slow-rapid changes): how long to repeat a new action, how long to take to consider and implement change ?

Then, he follows a course pursuit between means and ends, choosing and building each step, the context to choose (local optimization or not).

Variational rationality modelizes human behavior as a succession of worthwhile, aspiration driven and more or less goal oriented changes, entwinning more or less long periods of stability with periods of change (ambidextry and punctuated).

15 Conclusion

Further researchs will generalize our variational approach to group dynamics, the evolution of organizations and the dynamics of the relations between interrelated agents (synchronic or diachronic variational games), goal setting and goal striving with multiple objectives, gradual bargaining theory, time pressure and decision with bounded horizon where speed of decision and making matters (reativity costs), consideration set formation (choosing the context to choose), inexact proximal algorithms with convergence in finite time and error bounds, ...etc...This will help to give micro foundations to the behavioral theory of the firm and the resource-capability-competence- knowledge-evolutionary based theory of the firm. It will also provide a general model for content and process theories of motivation and inertia.

16 References

Abdellaoui Mohammed, Bleichrodt Han, Paraschiv Corina (2007) "Loss Aversion under Prospect Theory: A Parameter-Free Measurement". on line.

Alicia Kritsonis, "Comparison of Change Theories", International Journal of Scholarly Academic Intellectual Diversity, volume 8, number 1, 2004-2005.

Ajzen, I. (1991). The theory of planned behavior. Organizational Behavior and Human Decision Process, 50, 179-211.

Attouch.H- Soubeyran.A, (2010) "Local Search Proximal Algorithms as Decision Dynamics with Costs to Move", Set-Valued and Variational Analysis, ...

Bandura, A. (1989). "Human Agency in Social Cognitive Theory". American Psychologist, 44, 1175-1184.

Bandura. A. (1991). " Social cognitive theory of self regulation", Organizational Behavior and Human Decision Processes, 50, 248-287.

Bandura, A. (1997) Editorial: "The anatomy of stages of change". American Journal of Health Promotion, 12, 8–10.

Ada Aruffo and Gianfranco Bottaro (2010) "Some Variational Results Using Generalizations of Sequential Lower Semicontinuity",

Fixed Point Theory and Applications, Hindawi Publishing Corporation, Volume 2010, Article ID 323487, 21, pages doi:10.1155/2010/323487

H. Brezis, F. E. Browder. (1976) "A general principle on ordered sets in nonlinear functional analysis". Adv. Math. 21:355-364.

Chen. Y-Cho.Y and Yang. L (2002), " Note on the results with lower semi-continuity", Bull. Korean Math.Soc. 39, no 4, 535-541.

Chen.Yan and Robert Gazzale, (2004) "\When Does Learning in Games Generate Convergence to Nash Equilibria? The Role of Supermodularity in an Experimental Setting," The American Economic Review, 5 (94), 1505-1535.

Ekeland.I (1974) " On the variational principle", J. Math.Anal. Appl.47, 324-353.

Holland J.H. (1975), " Adaptation in natural and artificial systems", University of Michigan Press, Ann Arbor, MI.

Hoos, H.H. and Stutzle, T. (2005) "Stochastic Local Search: Foundations and Applications", Morgan Kaufmann.

Kahneman, D., Tversky, A. (1979). "Prospect theory: An analysis of decisions under risk". Econometrica 47 (2): 263–291. doi:10.2307/1914185.

Kotter John (1996) "Leading Change" by John Kotter. Harvard Business School Press.

Lewin K (1947) "Frontiers in Group Dynamics: Concept, Method and Reality in Social Science; Social Equilibria and Social Change".

Lewin, K, (1951), Field Theory in social Science, New York: Harper & Row.

Human Relations June 1947 1: 5-41.

Charles E. Lindblom (1959) "The Science of "Muddling Through", Public Administration Review, Vol. 19, No. 2, pp. 79-88

Lippitt, R., Watson, J. and Westley, B. (1958) "The Dynamics of Planned Change". New York: Harcourt, Brace and World.

Joseph T. Mahoney (2005) "Economic Foundations of Strategy", SAGE Publications, Inc

Paola Manzini Marco Mariotti (2008) "Categorize Then Choose: Boundedly Rational Choice and Welfare". on line

March. J.G, (1991) "Exploration and Exploitation in Organizational Learning", Organization Science, Vol. 2, No. 1, pp. 71–87.

Mennucci Andrea, (2007) " On asymmetric distances". Mimeo Scuola Normale Superiore Piazza Dei Cavalieri 7, 56126, Pisa, Italy

Nelson R-Winters S, (1982), "An Evolutionary Theory of Economic Change". Cambridge (Mass.), Belknap Press/Harvard University Press.

Prochaska, J.O. and Di Clemente, C.C. (1986). Towards a comprehensive model of change. In: W.R. Miller and N. Heather (Eds), Treating addictive behaviours: Processes of change. NewYork: Plenum Press.

B. Ranjbarian M. Kia (2010) "The Influence of Personality Traits on Consideration Set Size", European Journal of Social Sciences – Volume 15, Number 2, 262-269.

Richard M. Ryan and Edward L. Deci (2000) " Intrinsic and Extrinsic Motivations: Classic Definitions and New Directions", Contemporary Educational Psychology 25, 54–67; doi:10.1006/ceps.1999.1020, available online at http://www.idealibrary.com on.

Rogers, E.M. (1983). Diffusion of innovations. New York: Free Press

Rumelt, Richard P (1995). "Inertia and Transformation," in Montgomery, Cynthia A., ed., Resources in an Evolutionary Perspective: Towards a Synthesis of Evolutionary and Resource-Based Approaches to Strategy, Norwell, Mass.: Kluwer Academic Publishers, 1995, pp. 101-132.

Schumpeter, Joseph A. (1950), "Capitalism, Socialism and Democracy", London: Unwin.

Simon, H. A. (1956). "Rational choice and the structure of the environment". Psychological Review, Vol. 63 No. 2, 129-138

Simon, H. A. (1978). "Rationality as a process and product of thought". American Economic Review, 68, 1-16

M.V. Solodov · B.F. Svaiter (2000) "Error bounds for proximal point subproblems and associated inexact proximal point algorithms", Math. Program., Ser. B 88: 371–389.

Soubeyran.A (2010, a) " "Variational rationality, a theory of individual stability and change: worthwhile and ambidextry behaviors". Pre-print. one line.

Ume Jeong Sheok (2002) " A minimization theorem in quasi metric spaces and its applications", IJMMS 31:7. 443–447

PII. S0161171202110404, http://ijmms.hindawi.com, © Hindawi Publishing Corp.

Teece, D.; Pisano, G.; Shuen, A. (1997). "Dynamic capabilities and strategic management". Strategic Management Journal. Vol. 18, Iss. 7, 509-533

Oliver E. Williamson (1991) "Comparative Economic Organization: The Analysis of Discrete Structural Alternatives", Administrative Science Quarterly, Vol. 36, No. 2, pp. 269-296.

17 Appendix 1: The co determination of aspiration gaps and consideration sets

This appendix wants to show that aspiration gaps and consideration sets are determined jointly. The argument is that the consideration process is progressive, balancing between extensive search (a not so much serious examination of the utility of a lot of alternatives), and intensive search (a serious examination of few of alternatives). Then, consideration sets form progressively and the related aspiration level (an estimation of the supremum of the utility of alternatives over this set) becomes more and more precise. The larger the consideration set the less precise will be the estimation of the aspiration level.

What the agent have to consider each step Consider the simple case of actions $x \in X$ repeated one time, $t(x) = 1$ (see section 4) and, to save notations, forget the role of experience e. An action produces an outcome (performance) $Q(x) \in \mathbf{Q}$ and generates a cost $K(x) \geq 0$. The gross payoff related to this action is the benefit (revenue) $B(Q(x))$. The net payoff is the profit $g(x) = \Pi(x, Q(x))) = B(Q(x)) - K(x)$. In our approach, starting from having done before action $x \in X$ (the statu quo), the agent must, each step, consider several aspects,

i) imagines new actions $y \in F(x)$, and codifies their future execution as a succession of elementary operations $y = a^1oa^2o...a^{i-1}oa^ioa^{i+1}...oa^l$, using each some resources. The agent uses deletion $a^1oa^2o...a^{i-1}oa^{i+1}o...oa^l$, insertion $a^1oa^2o...oa^{i-1}oboa^{i+1}o...oa^l$, . and modification operations $a^l, a^1oa^2o...oa^{i-1}oboa^{i+1}o...oa^l$ operations to imagine new actions starting from old.

ii) estimates their outcome $Q(y)$, their benefit $B(Q(y))$, the cost to do them $K(y)$, their net payoff $g(y) = \Pi(y, Q(y)))$, their utility $U\left[g(y)\right]$, or their value (variational utility) $U\left[g(y) - g(x)\right]$.

iii) imagines and estimates the capabilities $\chi(y)$ to do each of them,

iv) imagines and evaluates the means $\omega \in \Omega(\chi(x), \chi(y))$ to pass from the present capabilities $\chi(x)$ to the future capabilities $\chi(y)$.

In a choice problem, the agent is dispensed to imagine new actions y and has not to imagine and estimate new capabilities $\chi(y)$ to do these new actions, nor the means ω . In a production problem like creation, invention, innovation, he does.

Each step, the formation of consideration sets is progressive. The agent first considers all these aspects in a rather vague way, ignoring a lot of ciriteria, or examining each criteria not very seriously. Then, he rejects some actions because of too low payoffs, or too difficult to build capabilities. Then, he considers more seriously the not rejected actions, ..; etc... This shows that, each step, the agent co-determines his aspiration level and his consideration set, after a lot of "allers-retours".

Consideration sets for choice problems Two stages decision processes. For choice problems they select a subset of alternatives, the consideration set, and the choice of the final alternative is from this reduced set, moving from a first stage incomplete and less effortful screening to a second stage much more serious and complete screening within the reduced set. The formation of consideration sets have been emphasized in Management Science to describe the behaviors of consumers (dealing with choice problems, see Ranjbarian-Kia, 2010 for a nice survey). Howard-Sheth (1969) define an evoked set, Wright and Barbour (1977) name it a consideration set and Brisoux-Laroche (1980) develop a model of consideration set divided into a foggy set (that includes all alternatives the agent knows exist, "yet the consumer does not have enough information, or enough desire, to seek out information about the brand and to evaluate the brand based on its attribute") and a processed set that includes alternatives that have been evaluated (mentally processed by the agent on at least one attribute), itself including an evoked set, a hold set and a reject set (subsets of brands which, after evaluation, the consumer would consider to purchase, feel neutral and reject). In our case we extend this construct to deal both with choice problems (without any creation of something) and production problems. Then awareness comes not only from information on some hidden action which pre-exists but also from imagination which considers new actions (thus enlarging the initial state space). Then, consideration becomes more and more serious, balancing more and more criteria.

ABS and CBS evaluation-elimination process. We will use the term criteria, characteristics, features, aspects, attributes as synonyms. Payne, Bettman, Johnson (1993) have emphasized that a decision maker can use two polar types of evaluation processes: i) a "characteristic-based search" (CBS) where the agent analyses the value of a single attribute of various alternatives before examining the next attribute, ii) an "alternative-based search" (ABS) where he analyses multiple attributes of a given alternative before examining the next alternative. Standard choice theoretical models are ABS. In our model, the consideration stage uses an extensive (in the large) CBS evaluation process, while the second stage uses an intensive (in depth) ABS evaluation process. The goal of a CBS process is to help to reject (consider again or not), and the goal of an ABS process is to help to accept (satisfice). The first can be far less costly than the second because not satisficing one criteria can be enough to reject. To accept an alternative requires to verify that several criteria are satisfied, while to reject an alternative, it is sufficient to verify that one criteria is not satisfied. In the same spirit, a disjunctive screening rule considers an alternative if one feature or set of features is above a threshold, a conjunctive screening rule considers an alternative if all of its features are above a minimum level and a compensatory screening rule weights criteria and adds their attribute levels, where high levels on some features can compensate for low levels on other features (Gilbride-Allenby, 2002). There are also subset conjunctive, disjunctive of conjonctions, lexicographic, elimination by aspects, and q-compesatory rules (Hauser-Ding-Gaskin, 2009).

A progressive way to evaluate alternatives is to rank criteria in ascending order of importance, relative to their costs of evaluation, and to start using the first criteria, eliminates alternatives, uses the second criteria, eliminates more alternatives within the remaining, ... This is a nested evaluation process. The inspection becomes more and more complete over less and less alternatives.

Consideration sets for production problems. In this case, where agents produce new things from old ones, consideration processes include also self efficacy and self evaluation not only of the capabilities of the agent (Bandura, 1991) but also of his dynamic capabilities. For an agent self efficacy is the beliefs about his capabilities. They influence how he sets his aspiration level, the higher the sense of efficacy, the higher will be the aspiration level. The same can be said about dynamic capabilities, the aspiration level being the highest goal level the agent beliefs he can reach by developping new capabilities. Much more can be said on that point which shows that aspiration levels and consideration sets are engaged in a course pursuit. Each period, given an initial aspiration level, the agent considers capabilities to approach this aspiration level. If these capabilities appear to be too costly to obtain, he will revise his aspiration level and so on. As Bandura (1991) emphasized, human self motivation relies on both discrepancy production (pro-ative control) and discrepancy reduction (reactive control).

18 Appendix 2: Motivation to change, goals as frames and emotions

Goal framing generates emotional preferences Setting goals (aspiration and satisficing levels) is a way to "focuse on" changing situations. It generates emotions which come most of the time from ex ante and ex post evaluations between old and new situations. This is the case when an agent starts from a given situation $(r, g(r/e)) \in \mathbf{R} \times \mathbb{R}$, where $g(x, e)$ is the payoff to perform one time more the temporary routine $r \in \mathbf{R}$, given his present experience $e \in \mathbf{E}$, which includes his added experience of having done r the period before. Suppose that the agent tries to satisfice a given satisficing level, $\widetilde{g}(r, e)$, $g(r/e) < \widetilde{g}(r, e) < \widehat{g}(r, e)$, based on his aspiration level $\widehat{g}(r, e)$ at r.The agent considers four gaps:

i) his aspiration gap, $\widehat{a}(r,e) = \widehat{g}(r,e) - g(r/e) \geq 0$, which generates an initial frustration feeling $-\lambda(r,e)\widehat{a}(r,e)$,where $\lambda(r,e) > 0$ is the intensity of his motivation. Either the agent, having a good morale, tries to reduce the aspiration gap $\widehat{a}(r,e)$ by trying to improve. If not, being depressed or discouraged, he renounces and will remain frustrated for ever, unless external causes change the situation. If he tries to improve, the agent estimates the utility $g(s,e)$ of doing a new temporary routine $s \in F(r,e)$ within his consideration set.

ii) his advantage to change, $a(r,s,e) = g(s/e) - g(r/e) \geq 0$, which generates a contentement feeling $\mu(r,e)a(r,s,e)$ where $\mu(r,e) > 0$ is the rate of contentment.

iii) his satisficing gap, $\widetilde{a}(r,s,e) = g(s/e) - \widetilde{g}(r,e) \gtreqless 0$ which generates a contentment or disappointment feeling $\nu(r,e)\widetilde{a}(r,s,e)$, $\nu(r,e) > 0$. Then, $\widetilde{a}(r,s,e) = a(r,s,e) - [\widetilde{g}(r,e) - g(r/e)] = a(r,s,e) - \beta(r,e)\widehat{a}(r,e)$ where $\widetilde{g}(r,e) = g(r/e) + \beta(r,e)\widehat{a}(r,e)$.

iv) his residual aspiration gap, $b(r,s,e) = \widehat{g}(r,e) - g(s/e) = \widehat{a}(r,e) - a(r,s,e)$ which generates a frustration feeing $\tau(r,e)b(r,s,e)$,where $\tau(r,e) > 0$ is the rate of frustration. Such a residual aspiration gap must be filled later.

Hence, the agent, uses low, high and medium frames, the statu quo value $g(r/e)$, which is the net utility to perform the temporary routine r a second time, the aspiration level $\widehat{g}(r,e)$, and the satisficing level $\widetilde{g}(r,e)$.Depending of how much he satisfices his goals, he feels different emotions (frustation contentment, disappointment, deception). Then, setting goals, the agent builds a motivation to change function which depends on all the strenghts of his emotion feelings

$M(r,s,e) = -\lambda(r,e)\widehat{a}(r,e) + \mu(r,e)a(r,s,e) + \nu(r,e)\widetilde{a}(r,s,e) - \tau(r,e)b(r,s,e)$.

Some manipluations gives

$M(r,s,e) = [\mu(r,e) + \nu(r,e) + \tau(r,e)]\,[a(r,s,e) - \eta(r,e)\widehat{a}(r,e)]$,

where $0 < \eta(r,e) = [\lambda(r,e) + \nu(r,e)\beta(r,e) + \tau(r,e)] \,/\, [\mu(r,e) + \nu(r,e) + \tau(r,e)] > 0$..

An emotional preference $M(r,s,e)$ weights differences in utility (gaps) with respect to various reference points (frames): a lower bound, the statu quo $g(r/e)$, an upper bound, the aspiration level $\widehat{g}(r,e)$, and a medium frame, the satisficing level $\widetilde{g}(r,e)$.

For the conception of goals as frames, see Heath-Larrick-Wu (1999).

More generally a motivation to change function is a non linear function of all these gaps,

$M(r,s,e) = U\left[\widehat{a}(r,e), a(r,s,e), \widetilde{a}(r,s,e), b(r,s,e)\right]$.

The function $g(s/e)$ can represent an utility, a performance, a payoff, while the gaps $\widehat{a}(r,e), a(r,s,e), \widetilde{a}(r,s,e), b(r,s,e)$ represent differences in utility, performance, or payoffs. Then, the function $U(.)$ puts non linear weights on variations in utilities, performances or payoffs.

Emotions drive the formation of satisficing ratios "Emotions may be considered as the affective evaluation of a difference between an expected and a realized reward" (Coricelli, Rustichini, 2009). A motivation to change function defines a variable emotional preference: the agent is motivated to change from r to s if his motivation function is non negative, $s \geq_e r \iff M(r,s,e) \geq$

$0 \Longleftrightarrow a(r,s,e) \geq \eta(r,e)\widehat{a}(r,e)$.This shows that, setting a satisficing ratio $0 < \beta(r,e) < 1$ such that $a(r,s,e) \geq \beta(r,e)\widehat{a}(r,e)$ will motivate the agent to change if $M(r,s,e) \geq 0$, ie if

$a(r,s,e) \geq \beta(r,e)\widehat{a}(r,e) \Longrightarrow a(r,s,e) \geq \eta(r,e)\widehat{a}(r,e)$ ie if $\beta(r,e) \geq \eta(r,e)$. This is true if

$\beta)\left[\mu+\nu+\tau\right] \geq \left[\lambda+\nu\beta+\tau\right]$

$\Longleftrightarrow \beta\left[\mu+\tau\right] \geq \lambda+\tau$ ie $1 > \beta \geq \left[\lambda+\tau\right]/\left[\mu+\tau\right]$.

where $\lambda = \lambda(r,r), \mu = \mu(r,e)$, ... etc....

This requires $\mu > \lambda$, ie that the positive feeling to improve dominates the negative feeling to have an unsatisfied gap of aspiration. This means that the agent prefers to do not renounce.An emotional preference weights differences in utility with respect to various reference points (frames): a lower bound, the statu quo $g(r/e)$, an upper bound, the aspiration level $\widehat{g}(r,e)$, and a medium frame, the satisficing level $\widetilde{g}(r,e)$.

An expectancy-valence formulation and prospect theory Consider for simplification the motivation to change function $M(r,s,e) = U\left[a(r,s,e)\right]$ where only one frame is considered, the statu quo $g(r/e)$.

Let $0 \leq \pi \leq 1$ be the probability to be able to move from r to s and $\psi(\pi)$ be a weight attached to this probability.Then, the expected utility to change from x to y is $\psi(\pi)U\left[a(r,s,e)\right] + (1-\psi(\pi))U\left[0\right] = \pi U\left[a(r,s,e)\right]$ where $a(r,r,e) = 0 \Longrightarrow U(0) = 0$.

This formulation is in the line of prospect theory (Kahneman-Tversky, 1979) which considers that the first element, values $U(.)$, are attached to changes in wealth (gains or losses) with respect to some reference point, for us $a(r,s,e)$, a gain if $a(r,s,e) \geq 0$ and a loss if $a(r,s,e) \leq 0$, rather than to final states, for us $g(s/e)$.The function $U(.)$ is zero at zero, increasing, concave for gains and convex for losses and describes a diminishing sensitivity towards an increase in gains or losses. The second element of prospect theory is a probability weighting function $\psi(\pi)$ that describes probability distortion by transforming given probabilities into decision weights

19 Appendix 3: Application to the Ekeland variational theorem

The Ekeland variational theorem (1974) Let (X,d) be a complete metric space, and let $f(.) : X \longrightarrow \overline{\mathbb{R}}$ be a lower semicontinuous from above and bounded from below function. Then, for each $\varepsilon > 0, \lambda > 0$ and $f(x_0) \leq \inf\left\{f(x), x \in X\right\} + \varepsilon$, there exists $x^* \in X$ such that i) $f(x^*) \leq f(x_0)$ and, ii) $f(x^*) < f(x) + (\varepsilon/\lambda)d(x^*,x)$ for all $x \neq x^*$.

Consider its supremum version. Let $g(.) : X \longrightarrow \overline{\mathbb{R}}$ be a bounded from above performance function. Its supremum is $\overline{g} = \sup\left\{g(x), x \in X\right\} < +\infty$. Let $f(x) = \overline{g} - g(x) = \widehat{a}(x) \geq 0$ be the aspiration gap at x.Then, the Ekeland theorem (1974) is equivalent to, Let (X,d) be a complete metric space, and let $g(.) : X \longrightarrow \overline{\mathbb{R}}$ be a upper semicontinuous from below and bounded from above function. Then, for each $\varepsilon > 0, \lambda > 0$ and $g(x_0) \geq \sup \{g(x), x \in X\} - \varepsilon$, there exists $x^* \in X$ such that i) $g(x^*) \geq g(x_0)$ and, ii) $g(x^*) > g(x) - (\varepsilon/\lambda)d(x^*, x)$ for all $x \neq x^*$.

The Ekeland theorem represents a specific case of our course pursuit model: solve, each step, the problem

$\overline{g}(x_n) = \sup \{g(y), y \in \Gamma(x_n)\}, x_n \in X, n \in N$, where actions are repeated only one time outside a behavioral trap, ie $r = r(x) = (x, 1)$, the worthwhile to change set $\Gamma(x) = \{y \in X, g(y) - g(x) \geq (\varepsilon/\lambda)d(x,y)\}$ coincides with the consideration set, the agent perfectly estimates the aspiration level, $\widehat{g}(x) = \overline{g}(x)$ for all $x \in X$, the capability space $\boldsymbol{\chi}$ is hidden, costs to change are equal to the distance between action (not the quasi distance between capabilities), the performance function is upper semicontinuous (and not variationally upper semicontinous).....

Let us give a self countained proof of the Ekeland variational principle in tem of worthwhile changes and aspirations gaps.

Definition 1 (Chen-Cho-Yang (2002):A function $g(.) : x \in X \longmapsto g(x) \in \overline{\mathbb{R}}$ is upper semi-continuous from below at $x^* \in X$ if $x_n \longrightarrow x^*, n \longrightarrow +\infty$ and $g(x_0) \leq g(x_0) \leq ... \leq g(x_n) \leq ....$imply $\lim_{n \longrightarrow +\infty} g(x_n). \leq g(x^*)$.

A function $g(.) : x \in X \longmapsto g(x) \in \overline{\mathbb{R}}$ is upper semi-continuous from below on X if it upper semi-continuous from below for every $x \in X$.

We will also use a path lower semi-continuity notion.

Definition 2: A function $\varphi(.) : x \in X \longmapsto \varphi(x) \in \mathbb{R}$ is path lower semi-continuous from above at $x^* \in X$ along a path of changes $\{x_n, n \in N\} \subset X$, which converges to x^* if $\lim_{n \longrightarrow +\infty} \varphi(x_n)$ exists and $\varphi(x^*) \leq \lim_{n \longrightarrow +\infty} \varphi(x_n)$.

The "Ekeland variational principle" as a "worthwhile to change aspiration principle". Let (X,d) be a metric space. Let $g(.) : x \in X \longmapsto g(x) \in \overline{\mathbb{R}}$ be a function bounded form above. Let $\overline{g}(x) = \sup \{g(y), y \in W(x)\} < +\infty$ and $\overline{a}(x) = \overline{g}(x) - g(x) \geq 0$ where $W(x) = \{y \in X, g(y) - g(x) \geq d(x,y)\} \subset X$. Then,

a) starting from every initial point $x_0 \in X$, there exists paths of worthwhile changes $x_{n+1} \in W(x_n), n \in N$ which "satisfice enough", $g(x_{n+1}) - g(x_n) \geq \theta \overline{a}(x_n), n \in N, 0 < \theta < 1$.

b) along every path of worthwhile changes which "satisfices enough", the aspiration gap $\overline{a}(x_n) \geq 0$ is non increasing and tends to zero, $\overline{a}(x_n) \longrightarrow 0, n \longrightarrow +\infty$.

c) every path of worthwhile changes is a Cauchy sequence which converges to a point $x^* \in X$ if X is complete.

d) Moreover, suppose that $g(.)$ is upper semi-continuous from below. Then,

d1) every path of worthwhile changes have an aspiration point $x^* \in W(x_n), n \in N$: it is worthwile to change from every x_n to x^*.

d2) the aspiration gap function $\overline{a}(.) : x \in X \longmapsto \overline{a}(x) \in \mathbb{R}$ is lower semi-continuous from above at $x^* \in X$ along a path of worthwhile changes $x_{n+1} \in W(x_n), n \in N$ which converges to $x^* : \overline{a}(x^*) \leq \lim_{n \longrightarrow +\infty} \overline{a}(x_n)$.

e) Then, starting from every initial point $x_0 \in X$, there exists a path of worthwhile changes which "satisfices enough" and converges to a trap $x^* \in X$ such that $\overline{a}(x^*) = 0$,and $W(x^*) = \{x^*\}$.

Proof. Point a) have been shown before. Point b) follows from $y \in W(x) \Longrightarrow W(y) \subset W(x)$. Then, $g(y) \geq g(x)$ and $\overline{g}(y) \leq \overline{g}(x)$ imply $0 \leq \overline{a}(y) \leq \overline{a}(x)$.Point c) follows from $x_{n+1} \in W(x_n), n \in N$. Then, $\{g(x_n)\}$ is not decreasing, $g(x_n) \longrightarrow g^*, n \longrightarrow +\infty$. and $x_q \in W(x_p)$ for $q \geq p$ (use the triangular inequality) . Then, the inequality $g(x_q) - g(x_p) \geq d(x_p, x_q)$ shows that for all $\varepsilon > 0$, it exists $p(\varepsilon) \in N$ such that $q \geq p(\varepsilon)$ implies $\varepsilon > g^* - g(x_p) \geq g(x_q) - g(x_p) \geq d(x_p, x_q)$.Then, every path of worthwhile changes is a Cauchy sequence which converges to a point $x^* \in X$ if X is complete. Consider point d1). Consider a sequence of worthwhile changes $x_{n+1} \in W(x_n), n \in N$ which converges to x^*.This implies, i) $x_q \in W(x_p)$ for $q \geq p$ (use the triangular inequality), ii) $g(x_{n+1}) \geq g(x_n), n \in N$ and iii) $g(x^*) \geq \lim_{n \longrightarrow +\infty} g(x_n) \geq g(x_q)$ for all $q \in N$ ($g(.)$ is upper semi-continuous from below). Then

$g(x^*) - g(x_p) \geq g(x_q) - g(x_p) \geq d(x_p, x_q) \geq d(x_p, x^*) - d(x_q, x^*)$. This implies

$g(x^*) - g(x_p) - d(x_p, x^*) \geq -d(x_q, x^*)$ where $d(x_q, x^*) \longrightarrow 0, q \longrightarrow +\infty$. This gives $g(x^*) - g(x_p) - d(x_p, x^*) \geq 0$ ie $x^* \in W(x_p), p \in N$.

Now, let us show point d2). From $y \in W(x) \Longrightarrow W(y) \subset W(x)$, we get $g(y) \geq g(x)$ and $\overline{g}(y) \leq \overline{g}(x)$, $0 \leq \overline{a}(y) \leq \overline{a}(x)$. Then, from d1) we have $W(x^*) \subset W(x_{p+1}) \subset W(x_p) \subset$ ....and $0 \leq \overline{a}(x^*) \leq \overline{a}(x_{p+1}) \leq \overline{a}(x_p) \leq$ ...This gives d2). Finally, from a) and b), starting from every initial point $x_0 \in X$, there exists a path of worthwhile changes which "satisfices enough" and converges to a trap $x^* \in X$ such that $\overline{a}(x_n) \longrightarrow 0, n \longrightarrow +\infty$.Then, $\overline{a}(x^*) = 0$.The inequality $y \in W(x) \Longrightarrow \overline{a}(x) \geq d(x, y) \geq 0$ implies $W(x^*) = \{x^*\}$.